Examples of Tasks from Course 2, Unit 5

What Solutions are Available?

Lesson 1: page 329, Modeling Task 2; page 331, Modeling Task 4; page 333, Organizing Task 3

Lesson 2: page 353, Modeling Task 1; page 354, Modeling Task 3; page 357, Organizing Task 4; page 359, Extending Task 2

These tasks are selected with the intent of presenting key ideas and skills. **Not every answer is complete**, so that teachers can still assign these questions and expect students to finish the tasks. If you are working with your student on homework, please use these solutions with the intention of increasing your child's understanding and independence.

As you read these selected homework tasks and solutions, you will notice that some very sophisticated communication skills are expected. Students develop these over time. This is the standard for which to strive. See Research on Communication.

The Discrete Mathematics page or the Scope and Sequence might help you follow the conceptual development of the ideas you see in these examples.

Main Mathematical Goals for Unit 5

Upon completion of this unit, students should be able to:

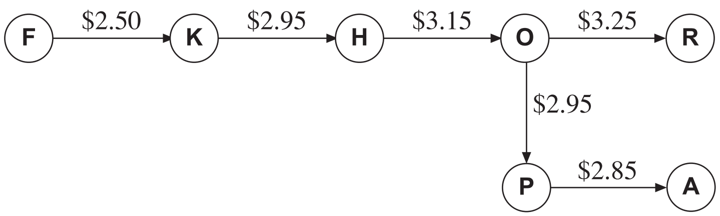
- model a variety of business and organizational problems with vertex-edge graphs
- develop the ideas of a minimal spanning tree, and shortest paths
- investigate algorithms to find optimal solutions to problems that involve efficiency, whether in terms of time, cost, or distance
- explore two classic problems: a Hamiltonian Circuit, and the Traveling Salesperson problem

Selected Homework Tasks and Expected Solutions

(These solutions are for problems in the book with 2003 copyright. If a student is using a book with an earlier copyright, you may notice that the problems don't match exactly, although the intent of the problems should be the same.)

Lesson 1, page 329, Modeling Task 2

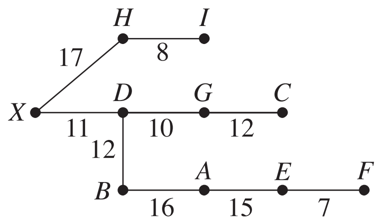
a. The table presents the same information as a graph with vertices representing the names and edges representing the costs of phoning. A spanning tree will have six edges. Use Kruskal's algorithm directly on the table to select edges of minimal cost and build a minimal spanning tree. The result is the network shown below and a total network calling cost of $17.65.



b. Answer left to students.

Lesson 1, page 331, Modeling Task 4

a. The minimum amount of wire needed is 108 feet. The following is one possible minimal spanning tree:



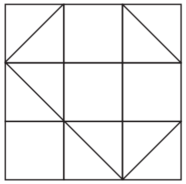
b. Answer left to students.

c. Answer left to students.

Lesson 1, page 333, Organizing Task 3

CPMP makes a deliberate effort to show students how different branches of mathematics are connected, sometimes using knowledge from one branch to develop understanding in another, when coordinate graphs are used to develop an understanding of the linear combination method of solving a system of equations, and sometimes revisiting an earlier problem with a new model. In this task, vertex-edge graphs are used to organize information about a geometric idea, rigidity.

a.
- Grid A is rigid. If students check each vertex and ask if it can be flexed, they are in fact checking if the vertex is part of a rigid triangle or not. Some of the vertices have been braced more than once (are part of two different triangles) and so some of the bracing may be redundant.
- Grid A, with three redundant braces removed is shown below. There are other possible solutions.

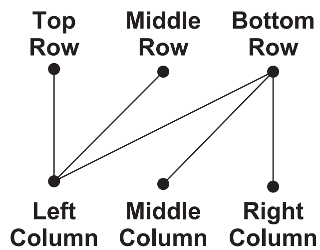


- Answer left to students.

b. Answer left to students.

c. Answer left to students.

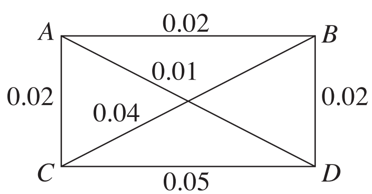
d. • One subgraph is shown below. After eliminating all "extra" edges, the remaining edges should be a spanning tree for the graph.



- Answer left to students.
- Answer left to students.

Lesson 2, page 353, Modeling Task 1

a.



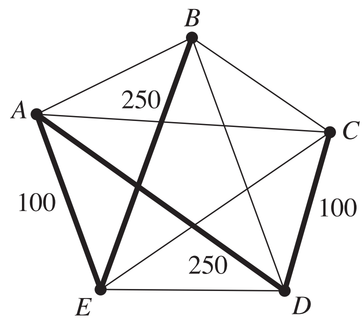
b. In the drilling problem, we have to visit each position once and return to the start; in the salesperson problem, the traveler had to visit each city once and return to the start.

c. *A*-*D*-*B*-*C*-*A* will give a circuit that visits each vertex once, total length 0.09. Other circuits are possible but none are shorter. (This is a Hamiltonian circuit.)

Lesson 2, page 354, Modeling Task 3

The information in the given matrix is about direct flights. For example, the cost for a direct flight from *A* to *B* is $500. But it is possible to connect *A*-*E*-*B* for $350. Therefore, students must make there own matrix with "shortest distances," meaning cheapest connections.

A graph might be easier to work with than the given matrix. On the graph below the bolded tree shows cheapest connections from *A* to any city.



A matrix showing these cheapest connections has been started below.

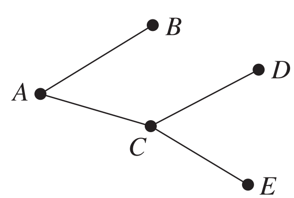
	A	B	C	D	E	Row Sum
A	–	350	350	250	100	1,050
B	350	–	200	300	250	1,100
C	350	200	–	100	250	900
D	250	300	100	–	350	1,000
E	100	250	250	350	–	950

The city with the smallest row sum would be the least expensive option for all travelers coming to that city.

Lesson 2, page 357, Organizing Task 4

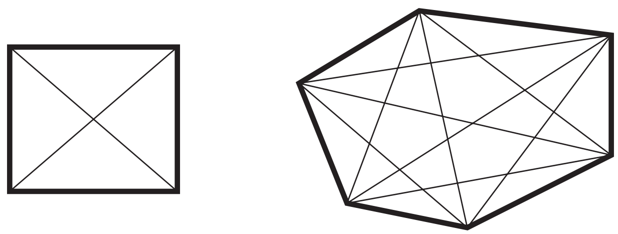
a. A tree cannot have a Hamiltonian path unless it has only vertices of degree one or two.

For example, for any path that arrives at *E*, the end of that branch of the tree cannot return to pick up other vertices.



This tree has vertices of degree 1 or 2 and does have a Hamiltonian path.

b. All complete graphs have a Hamiltonian circuit. For example:



c.
- The graph on the right has a Hamiltonian circuit (visits each vertex once and returns to start.)
- If there are the same number of vertices in each set, and this number is greater than one, then there is a Hamiltonian circuit. This is because in a bipartite graph you can only go back and forth between the two sets; there are no edges between vertices in the same set. So, when trying to find a Hamiltonian circuit, you start in one set, go over to the other, back to the first, and so on. This over-and-back movement implies that if you have the same number of vertices in each set, then you will be able to go over and back through all the vertices and end up where you started. Thus, there is a Hamiltonian circuit.

Lesson 2, page 359, Extending Task 2

Students have learned about several algorithms in this unit. They have also tried to invent their own. It is a mathematical habit of mind to search for routines that are efficient ways to solve problems. In some cases, they have found that there is no ***best*** *algorithm, just some that are more efficient than others.*