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CHAPTER 14

The *Core-Plus Mathematics Project*: Perspectives and Student Achievement

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The *Core-Plus Mathematics Project* (*CPMP*) curriculum is designed to make important and broadly useful mathematics meaningful and accessible to a wide range of students. The curriculum consists of a single core sequence for both college-bound and employment-bound students during the first 3 years of high school. This organization is intended to keep post-high school education and career options open for all students. A flexible fourth-year course continues the preparation of students for college mathematics. The completed curriculum is published under the title *Contemporary Mathematics in Context A Unified Approach* (Coxford, Fey, Hirsch, Schoen, Burrill, et al., 1997, 1998, 1999; Coxford, Fey, Hirsch, Schoen, Hart, et al., 2001).

This chapter provides a brief overview of the *CPMP* curriculum in terms of its design and theoretical framework, and a profile of the mathematical achievement of students who participated in the national field test of the curriculum.[1] Other focused research studies conducted in *CPMP* classrooms are reported elsewhere (cf. Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Kett, 1997/1998; Lloyd & Wilson, 1998; Truitt, 1998/1999; Walker, 199/2000).

The profile of *CPMP* student achievement continues to evolve over time. In Fall 1997, *CPMP* embarked on a longitudinal summative evaluation of the complete curriculum implemented with students who have completed middle school mathematics programs funded by the National Science Foundation and described elsewhere in this volume. Mathematical performance and attitudes of Fall 1997 high school freshmen have been monitored from Grade 9 through their first year of post-high school education or work. A summary of the findings of the longitudinal study is in progress.

BACKGROUND AND PERSPECTIVES

Each *CPMP* course was developed in consultation with an international advisory board, mathematicians, instructional specialists, and classroom teachers. In creating the *CPMP* three-year core curriculum (typically completed by students in Grades 9–11), the authors used a "zero-based" process (Mathematical Sciences Education Board, 1990) in which the inclusion of a topic was based on it own merits. In designing each course, we always asked and debated: "If this is the last mathematics students will have the opportunity to learn, is the most important mathematics included?" This approach resulted in the elimination or de-emphasis of some topics found in traditional curricula, a reordering of other topics, and the inclusion of what the authors believed to be the most broadly useful and important mathematical ideas. As a result, statistics, probability, and discrete mathematics have assumed a central position in each year of the curriculum. Course 4 formalizes and extends the core program with a focus on the mathematics needed to be successful in college mathematics and statistics courses.

In each year of the *CPMP* curriculum, mathematics is developed along interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of these strands is developed in focused units connected by common topics such as symmetry, functions, matrices, and data analysis and curve fitting. The strands also are connected across units by mathematical habits of mind such as visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, inventing mathematics, and providing convincing arguments and proofs. The strands are unified further by fundamental themes of data, representation, shape, and change. The choice of curriculum organization was influenced by the importance of connections among related concepts and procedures in developing deep understanding of mathematics (Skemp, 1987). This curriculum organization breaks down the artificial compartmentalization of traditional “layer cake” curricula in the Unites States and addresses weaknesses identified (Schmidt, 1998) in the findings from the Third International Mathematics and Science Study (TIMSS). In addition, developing mathematics each year along multiple strands capitalizes on the different interests and talents of students and helps to develop diverse mathematical insights (Hirsch & Coxford, 1997).

Tables 14.1 and 14.2 provide an overview of the scope and sequence of the *CPMP* 4-year curriculum. Recognizing that increasing numbers of college major programs involve the study of mathematics, though not necessarily calculus, *CPMP* Course 4 consists of a core of four units for all college-bound students, plus additional units supporting a variety of collegiate majors.

TABLE 14.1
The *CPMP* Core Curriculum

Unit No.	*Course 1*	*Course 2*	*Course 3*
1	Patterns in Data	Matrix Models	Multiple–Variable Models
2	Patterns of Change	Patterns of Location, Shape, and Size	Modeling Public Opinion
3	Linear Models	Patterns of Association	Symbol Sense and Algebraic Reasoning
4	Graph Models	Power Models	Shapes and Geometric Reasoning
5	Patterns in Space and Visualization	Network Optimization	Patterns in Variation
6	Exponential Models	Geometric Form and Its Function	Families of Functions
7	Simulation Models	Patterns in Chance	Discrete Models of Change
Capstone	Planning a Benefits Carnival	Forests, the Environment, and Mathematics	Making the Best of It: Optimal Forms and Strategies

TABLE 14.2
CPMP Course 4 Units

Core Units	Additional Units for Students Intending to Pursue Programs in: Mathematical, Physical and Biological Sciences or Engineering	Additional Units for Students Intending to Pursue Programs in: Social, Management, and Health Sciences or Humanities
1. Rates of Change	6. Polynomial and Rational Functions	5. Binomial Distributions and Statistical Inference
2. Modeling Motion	7. Functions and Symbolic Reasoning	9. Informatics
3. Logarithmic Functions and Data Models	8. Space Geometry	10. Problem Solving, Algorithms, and Spreadsheets
4. Counting Models		

Curriculum Development Principles

Several key principles guided the design of the *CPMP* curriculum. First and foremost is the belief that mathematics is a vibrant and broadly useful subject that should be explored and understood as an active science of patterns (Steen, 1990). As a result, experimentation, data analysis, and seeking and verifying patterns are pervasive in the curriculum. For instance, in Course 1 (Unit 2), *CPMP* students conduct experiments that simulate bungee jumping and analyze patterns in the relation between jumper weight and bungee cord stretch as a prelude to the study of algebraic expressions and equations. In Course 1 (Unit 7) they explore patterns in gender distribution of juries and multi-child families as an entree to the concepts and techniques of probability and statistics. In Course 2 (Unit 2) they study patterns in computer graphic images and then the related ideas of coordinate geometry, transformations, congruence, and similarity. An analysis of patterns in transportation and communication networks in Course 2 (Unit 5) leads to important concepts in graph theory that are widely used in computer and management sciences.

A second principle underlying the curriculum is that problems provide a context for developing student understanding of mathematics (Hiebert, et al., 1996; Schoenfeld, 1992). As suggested by the unit titles, mathematical modeling and related concepts of data collection, representation, interpretation, prediction, and simulation are emphasized.

Third, consistent with our view of mathematics as a science of patterns, exploration and experimentation necessarily precede and complement theory. Investigations are always accompanied by opportunities for students to analyze and abstract underlying mathematical structures that can be applied in other contexts and that, themselves, often are the subject of further investigations.

A fourth underlying principle is the incorporation of graphics calculators and project-developed calculator software as tools for developing mathematical understanding and for solving authentic problems. Graphics calculators permit the *CPMP* curriculum and instruction to emphasize multiple representations (verbal, numerical, graphical, and symbolic) and to focus on goals in which mathematical thinking is central. The use of graphics calculators promotes versatile ways of dealing with realistic situations and, for some students, reduces the manipulative skill filter that would have prevented them from studying significant mathematics.

The design of the *CPMP* curriculum was also informed by pedagogical principles, the most central of which is that classroom cultures of sense-making shape students' understanding of the nature of mathematics as well as the ways in which they can use the mathematics they have learned (Lave, Smith & Butler, 1988; Resnick, 1987, 1988;). Investigations of real-life contexts lead to (re)invention of important mathematics that makes sense to students and that, in turn, enables them to make sense of new situations and problems.

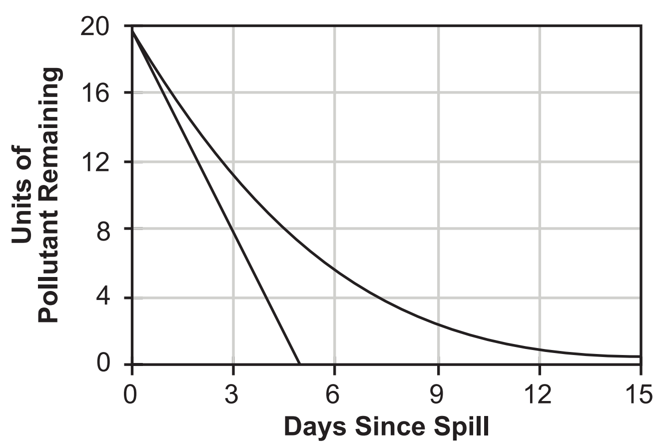
Integrated Instruction and Assessment

The *CPMP* curriculum was developed not only to reshape what mathematics all students have an opportunity to learn, but also to influence the manner in which learning occurs and is assessed. Each unit in the curriculum is developed around a series of four or five lessons in which major ideas are developed through student investigations of rich applied problem situations. Each lesson focuses on several interrelated mathematical concepts and often spans 4 or 5 days.

The *CPMP* instructional materials recognize the pivotal roles played by small-group collaborative learning, social interaction, and communication in the construction of mathematical ideas, particularly in cases involving females and underrepresented minorities. Each *CPMP* lesson is introduced by a problem situation which the entire class is asked to think about such as that in Figure 14.1. In this case, the context is an experiment that simulates pollution of a lake by some contaminant and the following clean-up efforts.

Think About This Situation

One of the problems of our complex modern society is the risk of chemical or sewage spills that can pollute rivers and lakes. Correction requires action of natural or human cleanup processes, but both take time. The graphs below show two possible outcomes of a pollution cleanup effort following an oil spill.



(a). What pattern of change in pollution level is shown by each graph?

(b). Which graph shows the pattern of change that you would expect from a pollution cleanup effort? Test your idea by running the pollution clean-up experiment several times and plotting the (*time, pollutant remaining*)

(c). What sort of equations relating pollution P and time t would you expect to match your plot of data? Test your idea using a graphing calculator or computer.

FIG. 14.1. Launching the Study of Exponential Decay (*Exponential Models* Unit, Course 1)

Once launched, a lesson usually involves students working together collaboratively in small groups or pairs as they investigate more focused problems and questions related to the launching situation. This investigative work is followed by a teacher-moderated class discussion in which students share mathematical ideas developed by their groups and together construct a common understanding of important mathematical concepts, methods, and approaches. Sharing, and agreeing as a class, on the mathematical ideas groups are developing is promoted by Checkpoints in the instructional materials. The sample Checkpoint in Figure 14.2 is the third of three Checkpoints in the lesson referenced in Figure 14.1. In Courses 1 and 2, students use *NOW-NEXT* language to describe linear and exponential patterns of change recursively. Each Checkpoint is followed by a related On Your Own assessment task to be completed individually by students.

√ Checkpoint

In this lesson, you have seen that patterns of exponential change can be modeled by equations of the form $y = a(b^x)$.

(a). What equation relates *NOW* and *NEXT* y values of this model?

(b). What does the value of a tell you about the situation being modeled? About the tables and graphs of (x, y) values?

(c). What does the value of b tell you about the situation being modeled? About the tables and graphs of (x, y) values?

(d). How is the information provided by values of a and b in exponential equations like $y = a(b^x)$ similar to, and different from, that provided by a and b in linear equations like $y = a + bx$?

Be prepared to compare your responses with those from other groups.

FIG. 14.2. Summarizing and Formalizing Mathematical Discoveries

Each lesson is accompanied by additional tasks to engage students in modeling with (M), organizing (O), reflecting on (R), and extending (E) their mathematical understanding developed through the investigations. MORE tasks are primarily for individual work outside of class.

Assessment is embedded in the *CPMP* curriculum materials and is an integral part of instruction. The instructional materials support continuous assessment of group and individual progress through observing and listening to students during the exploring and summarizing phases of instruction. In addition, there are individual assessments that measure understanding of mathematical concepts, methods, and skills. Each of the core courses also includes a thematic capstone as seen in Table 14.1. These project-oriented capstones provide rich mathematical problems whose solutions require the use of mathematics from each of the four strands studied during the year. This is an opportunity for students to review and consolidate their learning as well as demonstrate their mathematical growth over the year. For more detail on *CPMP* pedagogical perspectives and on the instructional model embedded in the curriculum, see Hirsch, Coxford, Fey, & Schoen (1995) and Schoen, Bean, & Ziebarth (1996).

SELECTED STUDIES OF STUDENT OUTCOMES

The *CPMP* evaluation has three interrelated goals: first, to provide a data-based guide for the development of the curriculum; second, to test the feasibility of the curriculum; and third, to identify strengths and weaknesses of the curriculum. Following the initial development and small-scale local tryouts of each course, the materials were pilot tested for 1 year, revised, and then field tested in the following year. The pilot test focused on providing data to guide the development of the curriculum.

Feasibility, strengths, and weaknesses were more the focus of the field test, although field-test data continued to guide the revisions of each course until publication. The feasibility of the *CPMP* curriculum was determined by how easily teachers and students could use the materials and by the impact of the materials on students' learning of important mathematical content. Strengths and weaknesses were addressed by measuring a variety of student outcomes and by analyzing those outcomes in comparison with those of students using more traditional curricula.

The *CPMP* evaluation model combines a large-scale field test with more focused research studies. In this chapter, results are drawn from the evaluation in order to paint a picture of students' mathematical achievement after completing each course in the *CPMP* curriculum. The outcomes for *CPMP* students are compared to those of students in more traditional mathematics curricula in some of the same field-test schools or to nationally representative norm groups.

Each of the first three *CPMP* courses was field tested in 36 high schools, and Course 4 was field tested in 28 high schools. Field-test schools were located in 11 states – Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. A broad cross-section of students from urban, suburban, and rural communities with ethnic and cultural diversity were represented.

The field-test schools were encouraged to include students with a wide range of achievement and interest in mathematics and, where possible, to group students heterogeneously. Limitations at local sites did not always make this possible. Approximately one-fifth of the Course 1 teachers reported that their classes included all ninth-grade students, including honors students. The most common *CPMP* class (as reported by 43.0% of the teachers) was composed of students with a wide range of prior mathematics achievement and interest; however, accelerated students were generally not included because they completed a traditional ninth-grade course in Grade 8 and continued with the next course in the sequence in Grade 9. Thus, accelerated students are probably underrepresented in the *CPMP* field-test sample

Comparative Studies of Courses 1 and 2

Design and Methodology

The goal of these studies was to compare the mathematical achievement of students experiencing the *CPMP* curriculum with that of students with similar mathematical aptitude and interest who were studying more traditional high school mathematics curricula. We administered some of the same pretests and end-of-year posttests to both *CPMP* students and to comparison students in more traditional curricula. The pretest provided a baseline for matching students in the two groups on their entering aptitude and the posttests served as outcome measures following the different curricular experiences.

Population

At the beginning of the first field-test year (1994-95), invitations were issued to lead teachers in each *CPMP* field-test school inviting them to identify, if possible, a set of students in more traditional ninth-grade mathematics courses in their school who were comparable in mathematical background and aptitude to their *CPMP* Course 1 students. For the first year, we envisioned comparison students coming mostly from ninth-grade algebra courses with some from Prealgebra. Thus, participation was limited to schools that offered both *CPMP* and a more traditional mathematics curriculum and that were willing to devote a class period in comparison classes for pretesting in September and two periods for posttesting in May or early June.

Eleven schools accepted the invitation. Six were from the Midwest, one urban, one rural and four suburban; three were from the West, one urban and two rural; one was urban and from the east; and the last was rural and from the South. At each site there were from two to five *CPMP* teachers and from one to three comparison teachers. The comparison classes for Course 1 study were composed of 20 Algebra, five Prealgebra, three General Mathematics, and two ninth-grade accelerated Geometry classes.

For the Course 2 study, we asked these 11 schools to posttest as many of the same comparison students as was feasible in May or June of the second year. This proved to be problematic for many schools because the comparison students had enrolled in a variety of mathematics classes in their sophomore year and were difficult to locate and to posttest at the end of the second year. As a result, only 5 of the 11 schools who had comparison groups in year 1 agreed to test their comparison students at the end of the second year—two suburban, Midwestern schools and three urban schools, one from the South and two from the West. The comparison students for the Course 2 study were a subset of the Course 1 comparison students and were enrolled in either Algebra, Geometry, or Accelerated Advanced Algebra.

Classroom Instruction

Because the *CPMP* curriculum and its teaching and assessment methods were new, at least one *CPMP* teacher from each school attended a 2-week workshop during the summer prior to teaching a *CPMP* course. In this workshop, teachers worked through the *CPMP* course materials by using a small-group investigative approach similar to that they would be using with their own students in the Fall. The comparison teachers had no special inservice program, because they were presumably accustomed to the curriculum and teaching method that they used.

Although what is reported are naturalistic studies occurring in classrooms with intact groups of students, the primary explanatory variable is the nature of the curriculum, including the curriculum-inspired pedagogy and assessment methods. The *CPMP* classes were composed of students with a wide range of entering mathematics achievement, including ninth-graders who normally would not have taken Algebra. Consequently, teachers commonly finished five or six of the seven units in Course 1, failing to complete Exponential Models in some cases and Simulation Models in nearly all cases. Survey data from 27 *CPMP* Course 1 teachers in these 11 schools indicate that a mean of 48.9% of class time (min. = 17%; max. = 80%) was spent in small group work. Approximately 80% indicated that calculators were available 100% of the time, and the most common restriction on calculator use was their unavailability for homework

The nature of the instruction in the comparison classes was not specified in advance, but at the end of the year comparison teachers described what transpired. Approximately 80% of the comparison teachers reported small group work was used either not at all or less than once a week. Approximately 74% of the comparison teachers reported that their students used a calculator more than once per week, although there are no data about how it was used. Understanding and solving linear equations in one variable was the main instructional goal for Course 1 comparison classes; teachers indicated a mean of 23% of the yearly class time spent on this topic and up to 50% of the time in some algebra classes.

Instruments

Two main instruments were used in the Course 1 and Course 2 comparative studies: first, a nationally standardized high school mathematics achievement test, and second, project-developed open-ended posttests for each *CPMP* course measuring important *CPMP* outcomes judged to overlap with goals of the comparison curricula.

Standardized Achievement. The standardized test was the mathematics subtest of the Iowa Tests of Educational Development (ITED), called Ability to Do Quantitative Thinking*, or* ITED-Q (Feldt, Forsyth, Ansley & Alnot, 1993). Rather than testing outcomes of a particular high school curriculum, the focus of ITED-Q is on quantitative thinking processes that are important for anyone with at least a high school education. In particular, the ITED-Q assesses high school students' ability to use appropriate mathematical reasoning in situations requiring the interpretation of data, charts, or graphs that represent information related to business, social and political issues, medicine, and science. It consists of three subtests.

1. Understanding Mathematical Concepts and Procedures (UMCP):
   These items require students to select appropriate procedures, make connections among various concepts, and identify examples and counterexamples of concepts.
2. Interpreting Information (Int. Inf.):
   These items require students to make inferences or predictions based on data or information often given in graphs or tables.
3. Solving Problems (Solve Probs.):
   These items require students to apply quantitative procedures to relatively novel situations, reason quantitatively, and evaluate reasonableness of solutions.

The mathematical content includes whole numbers, exponents, fractions, decimals, percents, ratios, geometry, measurement, estimation, rounding, statistics, probability, tables, and graphs. Although very little symbolic algebra is required, the ITED-Q is quite demanding for the full range of high school students. For example, on ITED-Q (Form K, Level 16) the median of the nationally representative norm group in Spring of Grade 10 is approximately 15 of 40 items correct, and the 99th percentile is approximately 35 of 40 items correct.

The ITED-Q correlates highly with other well-known measures of mathematical achievement, such as the Iowa Test of Basic Skills (ITBS), the ACT Mathematics test, and the Scholastic Achievement (SAT) Mathematics test.[2] A form of the ITED-Q was administered as a pretest to all *CPMP* and comparison students at the beginning of Course 1, and alternative forms as posttests at the end of Courses 1 and 2.

CPMP Posttests. In order to obtain a measure of students' attainment of specific curriculum objectives, the *CPMP* evaluation team developed open-ended achievement tests, called the Course 1 Posttest and Course 2 Posttest, each in two parts. Part 1 was designed to be a test of content that both *CPMP* and comparison students would have had an opportunity to learn that year, namely algebraic content for Course 1 and both algebraic and geometric content for Course 2. Part 2 of each *CPMP* Posttest included subtests of Data Analysis, Discrete Mathematics, Probability, and (in Course 1) Geometry; that is, content that the comparison students probably did not have the opportunity to study. Comparison students completed only Part 1 of the *CPMP* Posttest at the end of each year, and *CPMP* students completed both parts. These tests required students to construct their responses and to show and often explain their work. Only results from Part 1 are reported here.

Course 1 Posttest (Part 1) is composed of two contextual subtests, each requiring algebraic methods, and a third subtest of procedural algebra. The first two standards-oriented subtests require students to show that they understand algebraic concepts by applying them in realistic settings and interpreting their meaning in those settings. In particular, students translate between problem situations and algebraic representations, including linear equations and inequalities, tables, and graphs. These subtests also require students to rewrite algebraic expressions, solve equations that provide insights into the problem context, and explain how solutions or equivalent forms represent new information in the problem context. The third, more traditional subtest, requires students to solve linear equations in one variable and simplify linear expressions with no context.

The Course 2 Posttest (Part 1) also contains two contextual subtests, one algebraic and the other geometric, and a procedural algebra subtest. The contextual algebra and procedural algebra subtests are similar in design to their counterparts on the Course 1 Posttest but include some work with exponents and quadratic expressions. The geometry subtest presents a situation overlaid on a coordinate system; students are required to apply concepts and methods of coordinate geometry and explain the meaning of the results. Concepts and methods include finding an equation of a line given two points on it, the point of intersection of a vertical line and a second line, the midpoint of a segment, the distance between two points, an estimate of the area of an irregular closed region, and the reflection image of a figure across a given line. A related contextual problem requires the use of right triangle trigonometry to solve a triangle for the unknown length of a side.

Data Collection and Analysis

The lead teacher in each field-test school was responsible for testing at that site. Pretests were administered during regular class periods within the first 2 weeks of school and posttests during the last 2 weeks of school. Students had 40 minutes to complete the 40 item, multiple-choice ITED-Q, and 45 minutes to complete each part of the open-ended *CPMP* Posttests. Graphics calculators were allowed on all tests.

For the *CPMP* Posttests, a 5-point general scoring rubric was used:

- 4 for a “complete, correct response with clear unambiguous work or explanation”;
- 3 for a “good response with minor error of execution but not of understanding”;
- 2 for a “complete response showing understanding of some important ideas but misunderstanding of other ideas”;
- 1 for an “incomplete response that omitted important parts or included major errors”; and
- 0 for “no response or an irrelevant response.”

Graduate and advanced undergraduate Secondary Mathematics Education students were trained to use the rubrics to score the posttests. Training and practice on the scoring of each task continued until the inter-scorer agreement was 90% or higher. No scorers were aware of whether a particular test paper belonged to a *CPMP* or to a comparison student.

Matching CPMP and Comparison Groups

It makes sense to compare outcomes of groups whose previous achievement and aptitude were similar. To establish this similarity in the Course 1 study, the comparison students were separated into three subgroups according to whether they were just completing Prealgebra-General Mathematics, Algebra, or Accelerated Geometry. Each of these subgroups was matched to a subgroup of the *CPMP* Course 1 students. The matching variables were ITED-Q pretest score, school, and gender, in that order. If a match on pretest score could not be made with the same gender, then a student of opposite gender in the same school was chosen. If a match could not be made in the same school, a student of the same gender was chosen randomly from among potential pretest score matches in one of the other 10 schools. Only two comparison students could not be well matched in this way.

A similar process was used to match *CPMP* Course 2 students to the comparison students who were enrolled in Algebra, Geometry or Accelerated Advanced Algebra in Grade 10. All but seven comparison students were well matched with *CPMP* Course 2 students. As Tables 14.3 and 14.4 show, pretest means and standard deviations for matched groups were nearly identical.

Matching on one preachievement measure has the limitation that outcomes and characteristics not tested by the matching measure do not enter into the match. For example, the Accelerated Geometry students had completed 2 years of college-preparatory high school mathematics, whereas the *CPMP* Course 1 matched students were just completing their first high school mathematics course.

Matched Group Comparisons

ITED-Q Posttests. The ITED-Q Posttest (Form L, Level 15) results for the three matched groups of students in the Course 1 study are given in Table 14.3. The test developers provide standard scores for the entire test but not for the subtests, so means and standard deviations of the number of items correct on each subtest are presented. For the Prealgebra students and their *CPMP* matched sample, the mean scores of the *CPMP* students were significantly higher on the entire test, Interpreting Information Subtest, and Solving Problems Subtest. For the Algebra students and their *CPMP* matched sample, the mean scores of the *CPMP* students were significantly higher on the entire test and Interpreting Information Subtest. The accelerated Geometry students' ITED-Q and subtest means did not differ significantly from those of the matched group of *CPMP* students.

TABLE 14.3
ITED-Q Pretest and Posttest Information for Matched Groups of *CPMP* Course 1 and Comparison Students

		ITED-Q Pretest		*ITED-Q Posttest Total*		*UMCP (5 items)*		*Int. Inf. (18 items)*		*Solve Probs. (17 items)*	
Course	*n*	*Mean*	*SD*	*Mean*	*SD*	*No.*[a]	*SD*	*No.*[a]	*SD*	*No.*[a]	*SD*
Prealgebra	109	230.8	31.0	218.8	40.2	1.6	1.3	3.9	2.5	5.4	3.0
CPMP 1 (Match)	109	230.8	31.0	240.4*	35.5	2.0	1.4	5.1*	2.2	6.9*	3.0
Algebra	367	261.5	33.2	262.2	42.0	2.6	1.4	6.5	3.0	8.6	3.8
CPMP 1 (Match)	367	261.5	33.2	269.2*	37.9	2.7	1.2	6.9*	3.0	9.1	3.6
Acc. Geometry	49	294.6	22.1	304.0	21.6	3.6	1.0	9.4	3.0	12.6	2.6
CPMP 1 (Match)	49	294.6	21.8	299.1	23.7	3.6	0.8	9.3	3.1	11.6	2.9
All Comparison	525	258.3	36.1	257.1	46.2	2.5	1.4	6.2	3.3	8.3	4.0
All *CPMP* 1 (Match)	525	258.3	36.1	266.0*	39.5	2.7	1.3	6.8*	3.0	8.9*	3.6

Note. ITED-Q results were obtained by converting each student's raw score to a standard score using conversion tables for norm groups who took the test with a calculator. The standard score describes a student's location on an achievement continuum, regardless of the ITED-Q test form or the student's grade level. As an example, on the ITED-Q pretest (Form K, Level 15) the maximum standard score is 353, the minimum is 154, and the median is 255. The standard deviation is about 25 standard score points.

[a]The values in this column represent mean number of items correct.

*The t statistics for significant matched group mean comparisons in Table 3 are as follows: Prealgebra: ITED-Q Posttest Total, $t = -4.19$, $p < .001$; Interpreting Information, $t = -3.86$, $p < .001$; Solving Problems, $t = -3.80$, $p < .001$. Algebra: ITED-Q Posttest Total, $t = -2.38$, $p = .017$; Interpreting Information, $t = -1.96$, $p = .05$. All matched students: ITED-Q Posttest Total, $t = -3.37$, $p = .001$; Interpreting Information, $t = -2.80$, $p = .005$; Solving Problems, $t = -2.50$, $p = .013$.

The ITED-Q Posttest (Form K, Level 16) results for the three matched groups of students in the Course 2 study are given in Table 14.4. The results for the Algebra and Accelerated Advanced Algebra students should be viewed with caution because they are each based on a small number of students. With that caution, the available data show no significant differences in matched group means on the ITED-Q or any of its subtests.

TABLE 14.4
ITED-Q Pretest and Posttest Information for Matched Groups of *CPMP* Course 2 and Comparison Students

		ITED-Q Pretest		*ITED-Q Posttest Total*		*UMCP (7 items)*		*Int. Inf. (18 items)*		*Solve Probs. (15 items)*	
Course	*n*	*Mean*	*SD*	*Mean*	*SD*	*No.*[a]	*SD*	*No.*[a]	*SD*	*No.*[a]	*SD*
Algebra	31	227.4	29.2	248.6	22.0	2.7	1.2	5.4	2.3	5.0	1.7
CPMP 2 (Match)	31	227.3	29.2	252.0	33.2	2.6	1.4	6.2	2.4	4.9	2.7
Geometry	139	265.0	29.8	281.4	30.1	3.5	1.4	8.2	3.0	7.6	2.9
CPMP 2 (Match)	139	265.0	30.4	283.7	28.3	3.4	1.3	8.4	3.0	7.8	2.7
Acc. Adv. Algebra	25	291.9	17.6	311.4	17.5	4.6	1.5	11.3	2.5	10.4	2.5
CPMP 2 (Match)	25	291.8	17.5	305.0	23.8	4.4	1.5	10.6	2.8	9.8	2.5
All Comparison	195	262.5	33.4	280.0	32.3	3.5	1.5	8.1	3.3	7.5	3.0
All *CPMP* 2 (Match)	195	262.4	33.8	281.4	32.0	3.4	1.4	8.4	3.1	7.6	3.0

[a]The values in this column represent mean number of items correct.

It is important to note that the samples for the Course 1 and Course 2 studies are different from one another. The results in Table 14.3 are for one study and those in Table 14.4 are for another study with a different student sample. No inferences about 2 years of growth on the ITED-Q can be made from the two tables.

Table 14.5 contains data from all students in the five schools that had Course 2 comparison classes who completed the ITED-Q Pretest, the ITED-Q Posttest for Course 1, and the ITED-Q Posttest for Course 2. The 2-year trends in the means show that the comparison students started at a higher level on the pretest (59th compared with 46th national student percentile). The *CPMP* students grew 10 percentile points on the posttest administered after Course 1, compared to a 2-point increase for the comparison students. Both groups maintained their first-year increase in the second year.

Table 14.5
ITED-Q Means and Standard Deviations for All *CPMP* and Comparison Students

		ITED-Q Pretest			*ITED-Q Posttest (Course 1)*			*ITED-Q Posttest (Course 2)*		
Course	*n*	*Percentile*	*Mean*	*SD*	*Percentile*	*Mean*	*SD*	*Percentile*	*Mean*	*SD*
Comparison	186	59	264.7	35.3	61	272.5	43.9	61	281.6	33.8
CPMP	287	46	250.9	35.4	56	266.8	36.5	56	274.9	33.1

Note. Means are standard scores. Percentiles are national student percentiles. Data are for all students in the 5 schools with comparison classes who completed the ITED-Q pretest, the ITED-Q posttest for Course 1, and the ITED-Q posttest for Course 2.

A similar 3-year pattern emerges when the ITED-Q pretest and ITED-Q posttest means for Courses 1, 2, and 3 are analyzed for all 1,457 *CPMP* field-test students with complete data. There was no Course 3 comparison group. For students with 3 years of *CPMP*, means over the 3 years are compared with the growth of the nationally representative norm group's growth at the 62nd percentile, the pretest level of the *CPMP* students. As Figure 14.3 indicates, *CPMP* growth was strong in the first year, and the first-year increase was built on slightly in Course 2 and maintained in Course 3. This 3-year pattern is consistent, on average, in rural, urban, and suburban schools, for males and females, for various minority groups traditionally underrepresented in mathematics-related occupations, and for students for whom English was not their first language (Schoen, Hirsch & Ziebarth, 1998).

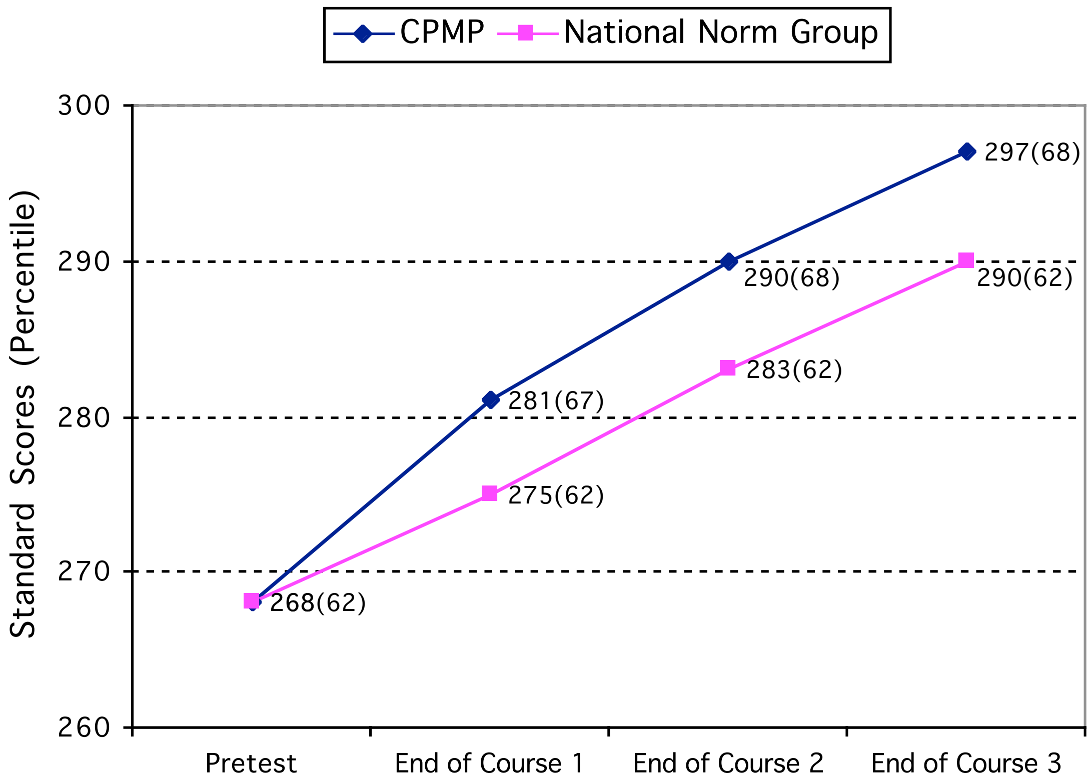


FIG. 14.3. Three-year trends in ITED-Q standard scores and national student percentiles (based on all 1,457 *CPMP* students with complete data).

In summary, the *CPMP* curriculum appears to have a positive effect on quantitative thinking as measured by the ITED-Q, with the greatest effect occurring in the first year of *CPMP* use. The particularly strong *CPMP* effect on the ITED-Q Interpreting Information and Solving Problems Subtests suggests that the impact of the curriculum is greatest in the areas of making inferences and predictions from data given in tables and graphs, applying quantitative procedures to relatively novel problems, reasoning quantitatively, and evaluating reasonableness of solutions.

It is not surprising that the largest mean differences occurred when *CPMP* students were compared with other non-accelerated students. Two findings that may seem surprising are that: the accelerated Geometry students' means were not significantly greater than those of the matched *CPMP* students in spite of the former's extra year of high school mathematics, and the positive effect of *CPMP* measured by the ITED-Q was mainly realized in the first year. The likely explanation for both of these results lies in the non-curriculum-specific content of the ITED-Q. Because the specific content of the extra year of traditional mathematics is not measured by the ITED-Q, the advantage of having that year is not reflected in the posttest scores. However, the curriculum-based experiences of reading, analyzing, and reasoning about problem situations enabled *CPMP* students to approach the dense verbal load of the ITED-Q as a sense-making activity. We conjecture that this learning occurred within the first year of *CPMP*, and that the further learning of more specific mathematics in future years of *CPMP*, as with the accelerated traditional students, may not be measured by the ITED-Q.

Course 1 CPMP Posttest. Although the matched groups were formed from the set of all students who had completed both the ITED-Q Pretest and the ITED-Q Posttest for each course, not all students in the matched groups completed the *CPMP* Posttests. Rather than make new matches, the following results are presented for all students in the matched groups who also completed the Course 1 *CPMP* Posttest. The groups formed in this way were still well matched on the ITED-Q Pretest.

The algebra strand of the *CPMP* curriculum emphasizes development of algebraic ideas through modeling of quantitative relationships in contextual problems, so it might be expected that *CPMP* students would perform well on such tasks. Similarly, the comparison students might be expected be fluent in algebraic symbolic manipulation with no context given. With some exceptions, the pattern of results in Table 14.6 aligns well with these expectations. Notice that mean differences on the contextual algebra subtests are approximately 0.5 to 1.0 standard deviation, favoring *CPMP* students. Mean differences on the procedural algebra subtests are approximately 0.5 standard deviation, favoring the comparison group for all but the Prealgebra students.

Table 14.6
Subtest Means and Standard Deviations for Matched Groups of *CPMP* Course 1 and Comparison Students

		ITED-Q Pretest		*Context. Alg. I*		*Context. Alg. II*		*Proced. Alg.*	
Course	*n*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Prealgebra	100	228.99	30.61	3.07	2.42	1.37	1.16	5.53	3.94
CPMP 1 (Match)	101	231.20	31.14	7.51*	4.04	2.95*	2.34	6.13	3.93
Algebra	338	262.76	32.47	6.98	4.07	3.43	2.25	11.90*	4.74
CPMP 1 (Match)	317	263.40	32.73	10.60*	3.99	4.56*	2.49	9.41	5.04
Acc. Geometry	47	295.00	22.44	9.60	4.14	4.23	2.29	14.89*	3.77
CPMP 1 (Match)	44	295.16	21.44	12.48*	3.33	5.91*	3.05	11.75	4.80
All Comparison	485	258.91	36.03	6.42	4.22	3.09	2.26	10.87*	5.32
All *CPMP* 1 (Match)	462	259.39	35.98	10.11*	4.20	4.34*	2.64	8.92	5.05

Note. Subtest means and standard deviations are on the Course 1 Posttest. Maximum scores are as follow: Contextual Algebra I, 16; Contextual Algebra II, 12; Procedural Algebra, 20.

*The t statistics for significant matched group mean comparisons are as follows. Prealgebra: Contextual Algebra I, $t = -9.45$, $p < .001$; Contextual Algebra II, $t = -6.06$, $p < .001$. Algebra: Contextual Algebra I, $t = -11.47$, $p < .001$; Contextual Algebra II, $t = -6.06$, $p < .001$; Procedural Algebra, $t = 6.51$, $p < .001$. Accelerated Geometry: Contextual Algebra I, $t = -3.64$, $p < .001$; Contextual Algebra II, $t = -2.98$, $p = .004$; Procedural Algebra, $t = 3.48$, $p = .001$. All matched students: Contextual Algebra I, $t = -13.44$, $p < .001$; Contextual Algebra II, $t = -7.84$, $p < .001$; Procedural Algebra, $t = 5.80$, $p < .001$.

A more detailed analysis of particular tasks may illuminate the pattern of results and suggest ways that curriculum developers can refine materials and teachers can modify implementation to improve student learning. As an illustration of the nature of the group differences in algebraic understanding and reasoning, the four-part task referred to in Table 14.6 as Contextual Algebra I is given in Figure 14.4. Means of the matched groups on each part of this task are also given. Numbers of students in the groups are the same as those given in Table 14.6.

In part (a) of this task, the intent was for students to indicate that 18 is the number of gallons of gasoline the boat had on board at the start and –2 indicates that 2 gallons of gasoline are used by the boat for each mile it travels. Such a response was given a score of 4. A score of 3 means either that both parts of the question were answered but with some vagueness such as “18 is the starting point" or "–2 is the slope" or that one question was answered at a 4 level and the other was vague or incorrect. A score of 2 was assigned if one part of the response was vague but relevant, that is, at the 3 level, but the other part was incorrect, or if one part of the question was answered at the 4 level but the other part was missing. Parts (b), (c), and (d) can be interpreted in a similar manner.

The number of gallons (y) of gasoline left in a large motor boat after traveling x miles since filling the tank is given by $y = 18 - 2x$.

	Prealg	*CPMP* Match	Alg.	*CPMP* Match	Acc. Geo.	*CPMP* Match
(a). Explain what 18 and –2 in the equation tell about the number of gallons.	1.1	2.2	2.1	2.8	2.6	3.2
(b). Graph this equation. Explain the role of 18 and –2 in the graph.	0.4	1.6	1.3	2.3	1.4	2.8
(c). After filling the gasoline tank, Helen drove the boat until there were 10 gallons left. How many miles had she driven? Explain how you can tell from the equation and how you can tell from the graph.	0.6	1.7	1.5	2.6	2.4	3.2
(d). How many gallons of gasoline were left after Helen had driven the boat 8 miles? Show or explain your work.	1.0	2.1	2.1	2.9	3.1	3.2

FIG. 14.4. Contextual Algebra I Task from the *CPMP* Course 1 Posttest (Means Based on 4 Points for Each Part)

The *CPMP* means are generally much higher than those of the matched comparison group. However, there is room for improvement by all student groups.

Course 2 CPMP Posttest. As in Course 1, rather than form new matched groups the following results are presented for all students in the original matched groups who also completed the Course 2 *CPMP* Posttest. The groups formed in this way were still well matched on the ITED-Q Pretest. Unfortunately, one of the five schools that administered the ITED-Q Posttest to comparison students in the Course 2 field test decided not to disrupt classes for a second day to administer the Course 2 *CPMP* Posttest to comparison students. This accounts for the large decrease in numbers in the geometry matched groups.

As indicated in Table 14.7, Course 2 *CPMP* students performed much better than matched comparison students on the contextual tasks and not as well on the procedural tasks, although the latter differences were only statistically significant in comparison to the Accelerated Advanced Algebra students who were completing their second full year of algebra.

Table 14.7
Subtest Means and Standard Deviations for Matched Groups of *CPMP* Course 2 and Comparison Students

		ITED-Q Pretest		*Coord. Geom.*		*Context. Alg.*		*Proced. Alg.*	
Course	*n*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Algebra	31	227.35	29.17	8.26	3.41	2.54	2.05	6.26	2.86
CPMP 2 (Match)	27	227.33	30.72	13.15*	3.75	7.04*	4.05	7.23	4.41
Geometry	67	261.52	30.76	10.97	4.19	3.99	2.83	7.51	3.19
CPMP 2 (Match)	69	260.72	33.05	16.84*	4.66	6.70*	3.99	7.23	4.41
Acc. Adv. Algebra	25	291.88	17.57	15.16	3.70	5.64	2.36	12.88*	3.43
CPMP 2 (Match)	18	289.50	16.48	17.33	4.33	8.83*	3.62	9.50	2.83
All Comparison	123	259.08	35.51	11.13	4.53	3.94	2.74	8.30	3.94
All *CPMP* 2(Match)	114	257.36	36.09	16.10*	4.70	7.14*	3.97	7.54	4.05

Note. Subtest means and standard deviations are on the Course 2 Posttest. Maximum scores are as follows: Coordinate Geometry, 24; Contextual Algebra, 12; Procedural Algebra, 16.

*The t statistics for significant matched group mean comparisons are as follows. Algebra: Coordinate Geometry, $t = -5.20$, $p < .001$; Contextual Algebra, $t = -5.43$, $p < .001$. Geometry: Coordinate Geometry, $t = -7.72$, $p < .001$; Contextual Algebra, $t = -4.56$, $p < .001$. Accelerated Advanced Algebra: Contextual Algebra, $t = -3.50$, $p = .001$; Procedural Algebra, $t = 3.42$, $p = .001$. All matched students: Coordinate Geometry, $t = -8.25$, $p < .001$; Contextual Algebra, $t = -7.27$, $p < .001$.

	Alg	*CPMP* Match	Geo.	*CPMP* Match	Adv. Alg.	*CPMP* Match
Solve $126 = 84 - 14x$. Explain your work and how to check it.	1.9	2.2	2.6	2.3	3.6	2.9
For the equation below, write an equivalent equation in the form $y = a + bx$. Explain how you are sure that the new equation is equivalent to the given one. $y = 52 + 20(x - 4)$	1.2	0.7	1.0	1.1	2.8	1.4
If $2^3 = 8$, $2^5 = 32$, and $(8)(32) = 2^n$, then $n =$ ____.	1.5	2.0	1.8	1.9	3.3	2.4
(0 - 2 points for each part)						
• Find $3(2^x)$ if $x = 4$.	1.1	1.2	1.5	1.2	1.9	1.7
• Does $3(2^x) = 6^x$ for any given x? Explain.	0.6	1.0	0.6	0.7	1.2	1.0

FIG. 14.5. Algebraic Procedural Tasks from the *CPMP* Course 2 Posttest (Means Based on 4 Points for Each Task)

The procedural algebra tasks from the *CPMP* Course 2 Posttest are given in Figure 14.5, together with means of the *CPMP* and comparison groups. Numbers of students in the groups are the same as those given in Table 14.7. Again, there is room for improvement for all groups of students, but differences in Course 2 group means are small. Consistent with the data presented in Table 14.7, means of the Accelerated Advanced Algebra group are greater than those of the *CPMP* matched group on the procedural algebra subtest; means of the Algebra and Geometry groups are similar to those of their *CPMP* matched group.

The results on the Course 1 and Course 2 Posttests are consistent with the differing emphases of the *CPMP* and traditional curricula. *CPMP* students probably had an advantage on the contextual problems as they were in a familiar form. However, individual interviews of *CPMP* and comparison students suggest that *CPMP* students are indeed better able to handle mathematical ideas in context and that *CPMP* students are less automatic and efficient with paper-and-pencil algebra procedures than some comparison students (Schoen, Hirsch & Ziebarth, 1998). The finding on contextual problem solving is consistent with extensive research evidence documenting the inability of many traditionally-educated students to move between symbolic and contextual situations and to solve verbally stated mathematics problems (Boaler, 1997; Schoenfeld, 1988, 1992).

Course 3 NAEP-based Achievement Results

Procedures

The National Assessment of Educational Progress (NAEP) is administered periodically to monitor U.S. students' achievement levels in various subject areas. In 1990 and 1992, a NAEP mathematics assessment was administered at several grades, including Grade 12. As another measure of *CPMP* students' achievement, a 30-item test was constructed by using released NAEP items from the five content categories and three process categories.

The NAEP-based test was administered in May 1997 to *CPMP* students at the end of Course 3. A total of 1,292 students in 22 *CPMP* field-test schools completed this test.[3] Six of the schools were urban, six were rural, and ten were suburban. In the presentation of the results, comparisons are made between the *CPMP* students and students in the nationally representative sample of 8,499 twelfth-grade students who took the NAEP in Fall 1990 or 1992 (Kenney & Silver, 1997).

The pattern of mathematics course-taking of the NAEP sample and the *CPMP* Course 3 group differed considerably. Students in the NAEP sample reported having taken the following mathematics courses: Calculus (10%), Precalculus (19%), Advanced Algebra (61%), Geometry (76%), or Algebra (87%). Thus, the sample included some students who had not enrolled in a mathematics course within the year prior to the NAEP testing and others who were taking calculus. In contrast, all of the *CPMP* students were just completing *CPMP* Course 3.

Testing conditions were not equivalent to the original administration of these same items by NAEP. For example, items were not administered in the same order, possibly affecting the item statistics. Further, the *CPMP* students had a graphics calculator available for the entire test; for the NAEP sample, a calculator was required for 11 of the 30 items and was not available for the others. Instead of a focus on comparing means, the item data from the NAEP sample are used as a benchmark of the difficulty of an item or of all items in a content, process, or NAEP-sample calculator category.

Results

The results from the NAEP-based test are given in Table 14.8. Of the five content categories, the mean percent differences between *CPMP* and NAEP sample students was greatest on data analysis, statistics and probability, a content strand of the *CPMP* curriculum that is not emphasized in most of the more traditional mathematics curricula.

Table 14.8
Mean Percent Correct and Mean Percent Differences by Group on NAEP-Based Test

Calculator, Content, or Process Type	*No. of Items*	*National (% correct)*	*CPMP 3 (% correct)*	*% Difference (CPMP–Nat.)*	*CPMP 3 SD*
			Content Categories		
Data, Statistics & Probability	4	44.5	67.0	22.5	24.5
Measurement	8	42.9	58.6	15.7	23.3
Algebra & Functions	6	41.5	53.2	11.7	21.8
Geometry	7	48.6	59.6	11.0	22.7
Numbers & Operations	5	34.1	43.8	9.7	25.7
			Process Categories		
Conceptual	10	44.3	60.8	16.5	18.4
Problem Solving	12	39.5	53.3	13.8	21.2
Procedural	8	45.4	55.6	10.2	23.4
			Calculator Access		
Calculator Available (All)	11	39.1	56.4	17.3	22.7
No Calculator (NAEP sample)	19	44.7	56.4	11.7	17.3
Total	30	42.7	56.4	13.7	17.9

Note. For *CPMP*, $N = 1{,}292$; for the national NAEP sample, $N = 8{,}499$.

Of the process categories, *CPMP* students' performance relative to the NAEP sample was best on conceptual items. This outcome is consistent with *CPMP*'s emphasis on sense making, applications, and problem solving with an accompanying deemphasis on procedural skill practice.

It might be expected that *CPMP* students would be relatively advantaged on the 19 items for which students in the national NAEP sample had no calculator available, but the data suggest the opposite. In fact, the difference between the performance of *CPMP* students and the national NAEP sample is greater on those items for which a calculator was permitted in the NAEP testing. Perhaps part of the explanation is that a graphics calculator is an essential tool in the *CPMP* curriculum, and students are taught its various uses as the need arises. Thus, *CPMP* students may become more proficient at using a calculator than students in many traditional classes in which the calculator often has a supplementary or enrichment role. This greater calculator proficiency is likely a positive factor for *CPMP* students on items for which a calculator is needed or potentially useful.

Sample tasks with percent correct for the *CPMP* and NAEP sample are presented in Figure 14.6 to illustrate the content, process and NAEP sample calculator-availability categories.

The NAEP-based test results show a pattern similar to the one that emerged from the ITED-Q and *CPMP* Posttest results. *CPMP* students demonstrate strengths in areas of conceptual understanding and problem solving in realistic contexts; they demonstrate somewhat lesser success with paper-and-pencil procedures and memory-based tasks.

Performance on College Entrance Examinations

Most universities and colleges require applicants to complete either the SAT or the ACT college entrance examinations, and the results are usually used in the admission process as one indicator of potential for success in college. Such examinations are important to both students and their parents.

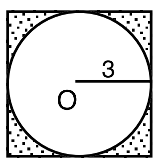
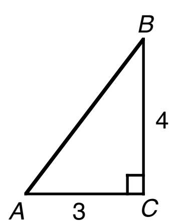
Sample NAEP Item	CPMP	NAEP
[*Data, Statistics & Probability, Problem Solving, NC*] From a shipment of 500 batteries, a sample of 25 was selected at random and tested. If 2 batteries in the sample were found to be dead, how many dead batteries would be expected to be in the sample? (a) 10 (b) 20 (c) 30 *(d) 40 (e) 50	80%	51%
 [*Measurement, Problem Solving, C*] In the figure above, a circle with center O and radius of length 3 is inscribed in a square. What is the area of the shaded region? (a) 3.86 *(b) 7.73 (c) 28.27 (d) 32.86 (e) 36.00	64%	37%
[*Algebra & Functions, Conceptual, C*] For what value of x is $8^{12} = 16^x$? (a) 3 (b) 4 (c) 8 *(d) 9 (e) 12	82%	34%
 [*Algebra & Functions, Procedural, NC*] In right triangle *ABC* above, cos *A* = *(a) $\frac{3}{5}$ (b) $\frac{3}{4}$ (c) $\frac{4}{5}$ (d) $\frac{4}{3}$ (e) $\frac{5}{3}$	34%	30%
[*Geometry, Conceptual, NC*] Which of the following is NOT a property of every rectangle? (a) The opposite sides are equal in length. (b) The opposite sides are parallel. (c) All angles are equal in measure. *(d) All sides are equal in length. (e) The diagonals are equal in length.	86%	71%
[*Number & Operations, Procedural, C*] A savings account earns 1 percent per month on the sum of the initial amount deposited plus any accumulated interest. If a savings account is opened with an initial deposit of $1,000 and no other deposits or withdrawals are made, what will be the amount in this account at the end of 6 months? (a) $1,060.00 *(b) $1,061.52 (c) $1,072.14 (d) $1,600.00 (e) $6,000.00	36%	15%

FIG. 14.6. Sample NAEP-Based Items of Various Types with Average Percent Correct by Group (C means students in the NAEP sample used a calculator on that item and NC that they did not.).

The SAT college entrance examination (SAT I) comprises two subtests, Verbal and Mathematics. The SAT Mathematics test measures mathematical reasoning and symbol sense, drawing on content from arithmetic, algebra and geometry. It requires understanding of basic algebraic and geometric concepts typical of the first 2 years of traditional high school mathematics but measures little standard paper-and-pencil algebraic symbolic manipulation. The scores on both subtests are standardized with a mean of 500 and standard deviation of 100.

The ACT college entrance examination consists of four subtests: English, Mathematics, Reading, and Science Reasoning. The ACT also reports a Composite score, the average of the four subtest scores. The ACT Mathematics subtest measures achievement on the content of the traditional college preparatory mathematics curriculum, including topics from elementary algebra, intermediate algebra, coordinate geometry, plane geometry and trigonometry. ACT Mathematics items test algebraic, geometric and trigonometric concepts and procedures and standard word problems usually intended to be solved by using equations or inequalities. ACT subtest and composite scores are reported on a scale ranging from 1 to 36.

Both the ACT and SAT allow, but do not require, students to use graphics calculators. Virtually no statistics, probability, or discrete mathematics are tested on either test.

Comparative Studies

One field-test school, referred to by the pseudonym Southeast High School, provided eighth-grade ITBS scores as baseline data. This school is in a suburban district in the South in which families are primarily middle to upper-middle income and parents are well educated. Approximately 89% of students are Caucasian and others are from various minority groups. Three years of high school mathematics are required for graduation.

At the beginning of the *CPMP* field test, Fall 1994, all students who qualified for Prealgebra or nonhonors Algebra were randomly assigned to *CPMP* Course 1 or to the appropriate Prealgebra or Algebra class. Many of these students completed Advanced Algebra or *CPMP* Course 3 in their junior year and took the SAT either in Spring or Summer of their junior year or in Fall of their senior year. Results for this set of *CPMP* and traditional students are given in Table 14.9. Both groups of students were comparable in mathematics achievement at the beginning of Grade 9. The difference between SAT Mathematics means is not statistically significant at the 0.05 level for students studying from the two different curricula.

Table 14.9
ITBS and SAT Information for *CPMP* and Traditional Students

		ITBS Math (Percentile)		*SAT Math*	
Group	*n*	*Mean*	*SD*	*Mean*	*SD*
CPMP	54	57.1	20.4	484.6	53.8
Traditional	44	57.5	17.9	467.0	67.5

Note. Information is for students at Southeast High School.

One urban school district in the Midwest provided ACT Mathematics and ACT Composite scores along with sixth-grade California Achievement Test (CAT) scores. This district does not give standardized mathematics tests to students in grades 7 or 8. The pseudonym, Midwest High School, is used, although the data come from two similar high schools in the same district in a city with a population of several hundred thousand. The students come from a mix of socioeconomic backgrounds, and approximately 30% of students are either Hispanic or African American. Midwest High School requires 3 years of high school mathematics for graduation.

At the beginning of the *CPMP* field test (Fall 1994), all students who qualified for remedial mathematics through algebra were randomly assigned by computer to *CPMP* Course 1 or to the appropriate traditional class. Many of these students completed Advanced Algebra or *CPMP* Course 3 in their junior year and took the ACT either in spring or summer of their junior year or in Fall of their senior year. Results for these students are given in Table 14.10. Students in the two curricula had comparable mathematical backgrounds at the beginning of high school. The ACT Mathematics mean for the *CPMP* group was almost identical to that of the traditional group.

In both Southeast and Midwest High Schools, the *CPMP* and traditional students were well-matched on before high school standardized mathematics test means. At each site, the only apparent systematic difference is that one group learned high school mathematics in the context of the *CPMP* curriculum and the active learning environment it promotes. Class observations, student and teacher interviews, and teacher surveys provide evidence that teachers in both schools implemented *CPMP* in a way that was well aligned with the project team's recommendations. Together the SAT and ACT Mathematics results suggest that well-matched *CPMP* and traditional students do not differ significantly on these college entrance examinations.

Table 14.10
Sixth-Grade Information for *CPMP* and Traditional Students

		CAT Math (Percentile)		*ACT Math*		*ACT Composite*	
Group	*n*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
CPMP	71	66.3	24.2	18.3	3.1	20.3	3.6
Traditional	42	68.5	26.4	18.4	3.5	19.1	3.6

Note. Information is for students at Midwest High School.

ACT Trends

By Fall 1999, eight *CPMP* field-test schools had adopted *CPMP* for all their Grade 9–12 students. Seven of the eight schools are midwestern schools where most of the students who plan to attend college take the ACT. Three of the eight schools are rural, two are urban, and three are suburban. Seven of the eight schools were also pilot-test schools, so beginning in 1995–1996 nearly all their students (mainly juniors) who took the ACT college entrance examination had experienced *CPMP* as their high school mathematics program. By 1996–1997, all students in all eight schools had experienced *CPMP* rather than the traditional mathematics program that had been in place in previous years.

The trend data in Table 14.11 are compiled from the 1997–1998 ACT school reports from these eight schools. ACT Science Reasoning scores are reported because this subtest requires reasoning in contextual, quantitative and graphical settings. The decrease in the number of *CPMP* students taking the ACT in 1997–1998 was due to the opening of a new high school in the town with the largest of the eight schools, thereby cutting the enrollment in the *CPMP* school by nearly one half. National means are those of all students who took the ACT each year, a number that has steadily increased over the 5 years from 891,714 in 1993–1994 to 995,039 in 1997–1998.

The all-*CPMP* schools may differ demographically from those in the national ACT sample, so the magnitude of a mean difference on a particular subtest at any given time is of little interest. However, a comparison of the annual trends for the *CPMP* students and the national ACT sample is informative. First, mean scores in the *CPMP* schools nearly all held steady or increased since 1995–1996 when examinees in those schools first completed *CPMP* rather than a traditional program. Second, the annual ACT Mathematics trends in the *CPMP* schools are similar to those in the ACT national sample. ACT Mathematics means in *CPMP* schools increased by 0.3 points from 1993–1994 to 1997–1998 compared with 0.6 in the national ACT sample. Over the same period, ACT Science Reasoning and ACT Composite means increased more in the all-*CPMP* schools than in the national sample (increases of 0.6 and 0.5, respectively, compared with an increase of 0.2). Although student populations may have changed in ways that affect ACT scores over this 5-year period, these longitudinal trends are consistent with a conclusion that the *CPMP* curriculum prepares students for the ACT at least as well as traditional mathematics curricula.

Table 14.11
Five-year Trends of ACT Means

		ACT Mathematics		*ACT Science Reasoning*		*ACT Composite*	
Year	*CPMP N*	*CPMP*	*National*	*CPMP*	*National*	*CPMP*	*National*
1993–1994	1,067	21.3	20.2	22.2	20.9	21.9	20.8
1994–1995	1,107	21.8	20.2	22.4	21.0	22.1	20.8
1995–1996	1,040	21.9	20.2	22.7	21.1	22.3	20.9
1996–1997	1,150	21.5	20.6	22.6	21.1	22.3	21.0
1997–1998	971	21.6	20.8	22.8	21.1	22.4	21.0

Note. ACT means are across 8 schools using the CPMP curriculum with all students beginning in either 1995–1996 or 1996–1997.

Performance on a College Mathematics Placement Test

A mathematics placement test that is currently used at a major university was administered to students in field-test schools in May 1999 at the end of *CPMP* Course 4 and at the end of traditional Precalculus. The university's mathematics department uses this placement test, compiled from a bank of items developed by the Mathematical Association of America, to make recommendations to entering freshmen concerning the college mathematics course in which they should enroll. This test contains three subtests—Basic Algebra (15 items), Advanced Algebra (15 items), and Calculus Readiness (20 items). The first two subtests consist almost entirely of symbolic manipulation tasks such as simplifying and factoring algebraic expressions, solving equations and inequalities, and finding equations for lines given sufficient conditions. The third subtest measures some of the important concepts and processes that underlie calculus, such as logarithmic and exponential equations, trigonometric functions and identities, composition of functions, rational functions and their domains, systems of nonlinear equations, and area of a rectangle under a curve. A graphing calculator (that does not do symbolic manipulation) is allowed on this test.

The *CPMP* Course 4 students included in the comparison that follows are all those in the 1998–1999 Course 4 field test who completed at least six of the seven "preparation for calculus" units of Course 4 as the last course in their sequence of *CPMP* Course 1–4. The Precalculus students, also from field-test schools, completed a traditional Precalculus course to end a sequence of Algebra, Geometry and Advanced Algebra. The two groups were further restricted to those students who indicated on a written survey their intention to attend a four-year college or university in the next school year. Both groups are composed of students who fell mainly in the 75th to 95th national percentiles, on average approximately 85th, on standardized mathematical achievement tests at the beginning of high school. Means and standard deviations by group and subtest are reported in Table 14.12.

Table 14.12
Placement Subtest Information for *CPMP* and Traditional Precalculus Students

		Beginning Algebra		*Intermediate Algebra*		*Calculus Readiness*	
Group	*N*	*Mean*	*SD*	*Mean*	*SD*	*Mean*	*SD*
Precalculus	177	11.4	2.3	9.6	3.2	10.5	4.3
CPMP 4	164	11.5	2.6	9.2	3.4	12.9*	4.7

*The t statistics for the significant Calculus Readiness group mean comparison are $t = -4.93$, $p < .001$.

The *CPMP* Course 4 mean is significantly greater than the Precalculus mean on the Calculus Readiness subtest, whereas the group means do not differ significantly on the Basic Algebra and Advanced Algebra subtests. This is more content-specific evidence to combine with the SAT and ACT findings presented earlier that *CPMP* students are at least as well prepared for entering college mathematics as students from more traditional curricula. An area of particular strength for the *CPMP* Course 4 students is understanding of the concepts and processes that underlie Calculus.

Performance in College Mathematics Courses

The first students who have experienced the entire four courses of *CPMP* in the field-test version entered college in Fall 1999. However, some preliminary evidence on how high school graduates who experienced the *CPMP* curriculum in its pilot version perform in collegiate mathematics courses is available. Freshmen mathematics course grade data for each year from 1995-1996 through 1998-1999 were gathered for all graduates of two similar high schools in one midwestern, suburban school district who enrolled at the University of Michigan. For purposes of this report, the pseudonyms East High School and West High School are used. Both East and West High School's 1995 and 1996 graduates experienced a traditional high school college-preparatory mathematics program with offerings through AP Calculus. This program continued at West High School. At East High School, all 1997 graduates who were not in an accelerated mathematics program and all 1998 graduates completed the *CPMP* pilot curriculum. Accelerated students among 1998 East graduates took AP Calculus as seniors after completing *CPMP* Courses 1 through 4 in previous years.

Located in a suburb with many affluent, well-educated residents, East and West High School buildings (enrollments of about 840 and 1,070, respectively) are 2 miles apart and demographically similar. Many adults in the community are professionals in upper management positions. Over 80% of the students are White, with Asian Americans comprising the largest of several minority groups. Fewer than 10 students in each school are eligible for the free lunch program. Freshman college mathematics course grades of graduates from these two schools who matriculated at the University of Michigan were analyzed by using computer data files with school names, but no student names, attached. Thus, the form of the data precludes any connecting of data to individual students, but it allows for analysis of 4-year school trends in college mathematics course-taking and grades.

Pertinent mathematics courses at the University are Precalculus, Calculus I, Calculus II, Calculus III, Introduction to Differential Equations, and honors (all honors math courses open to freshmen). Precalculus is the most basic mathematics course offered. Typically, freshmen enrolled in precalculus have completed 3 to 4 years of college-preparatory high school mathematics but not AP Calculus. Freshmen enrolled in Calculus I in fall semester have usually completed at least 4 years of high school mathematics through Precalculus or *CPMP* Course 4, and some may have taken a high school AP Calculus course. Spring-semester Calculus I classes include some students who successfully completed Precalculus in the Fall semester. Freshmen in Calculus II or Calculus III are placed there mainly because of high AP Calculus Examination scores or success in the preceding college calculus course in fall semester. Freshmen with exceptionally strong high school mathematics backgrounds and high AP Calculus Examination scores may take Calculus III in the Fall semester and Differential Equations in the Spring semester.

Table 14.13 gives the number of matriculants at the University of Michigan among the 1995, 1996, 1997 and 1998 graduates of East and West High Schools, the numbers of these graduates completing each mathematics course in their freshman year, together with grade point averages, and course averages. The grade point averages were calculated by using the University's system as follows: A+ (4.3), A (4), A- (3.7), B+ (3.3), B (3), …, D (1), D- (0.7), E+ 0.3), and E (0).

University mathematics course grades of East High School graduates for 1997 and 1998, when the *CPMP* pilot curriculum was in place, are generally higher than both pre-*CPMP* (that is, 1995 and 1996) East graduates and 1997 and 1998 West High School graduates. The number of 1997 and 1998 East High School graduates matriculating at the University of Michigan is greater than for the previous 2 years. As for courses in Calculus I and above, school trends in grade point averages (GPA) are shown in Figure 14.7.

Table 14.13
College Mathematics Mean Grade-Point Averages by School, Course, and Year

	East High School				*West High School*			
College Class	*1995 (50)*	*1996 (74)*	*1997 (87)*	*1998 (72)*	*1995 (34)*	*1996 (57)*	*1997 (45)*	*1998 (35)*
Precalculus	3.18	2.29	2.74	2.98	1.46	3.00	2.60	2.97
	(4)	(6)	(13)	(6)	(7)	(4)	(5)	(3)
Calculus I	2.86	2.60	3.08	2.89	2.33	2.82	2.58	2.87
	(14)	(19)	(32)	(25)	(7)	(13)	(15)	(7)
Calculus II	2.67	3.33	3.17	3.49	2.45	3.21	2.63	2.29
	(14)	(12)	(19)	(12)	(6)	(18)	(8)	(8)
Calculus III	2.66	3.10	2.95	2.99	2.50	3.17	3.34	2.34
	(5)	(4)	(6)	(8)	(2)	(11)	(6)	(5)
Intro. to Diff. Equ.	2.15	4.00	4.00	3.30	—	3.67	3.65	—
	(2)	(1)	(2)	(2)		(3)	(2)	
Honors	—	3.28	—	—	3.30	3.77	4.23	—
		(5)			(1)	(3)	(4)	
All Courses	2.76	2.89	3.06	3.07	2.15	3.15	2.92	2.57
	(39)	(47)	(72)	(53)	(23)	(52)	(40)	(23)

Note. Number of students is shown parenthetically; years denote year of graduation. East High School used *CPMP* in 1997 and 1998; West High used traditional curricula in all 4 years.

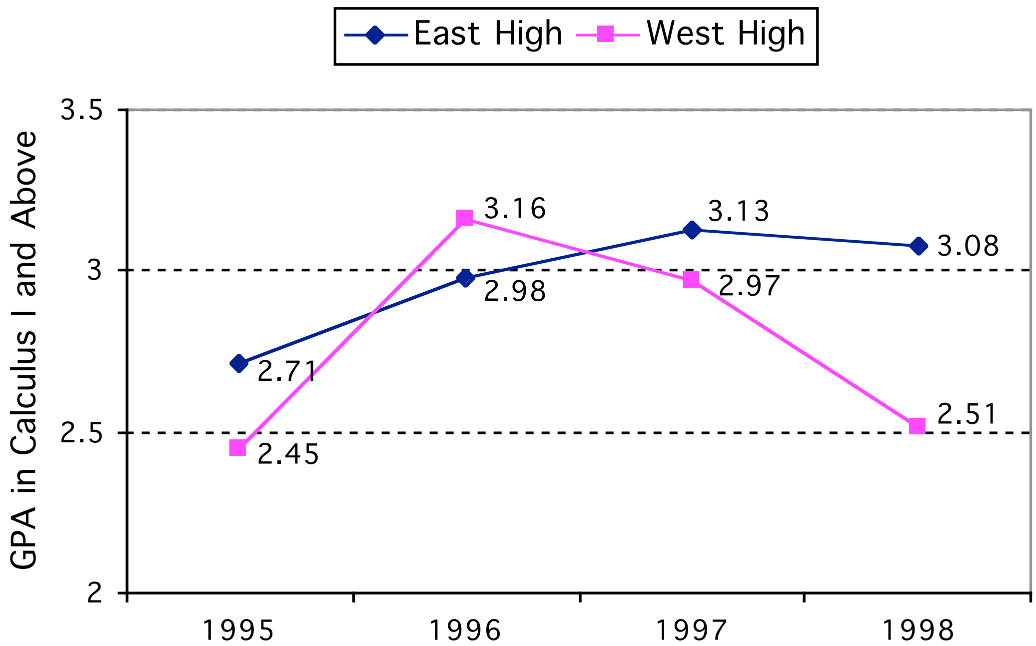


FIG. 14.7. Mean grade point averages in Calculus I courses and above.

While West High's grade averages varied greatly from year to year, those of East High were higher in 1997 and 1998 after using the *CPMP* curriculum than in the previous two years when a traditional curriculum was in place. The percent of course enrollments in Calculus I and above is greater for West High School in 1997 (88% compared with 82% for East High); but in 1998 when all East High graduates had completed the CPMP curriculum, these percents were 89% for East High and 87% for West High. More information concerning the relationship between individual students' high school and college mathematics records would allow for a more detailed interpretation of these data. However, this preliminary evidence suggests that students who experienced the pilot CPMP curriculum were at least as well prepared for calculus (AP or college level) as students in a more traditional curriculum.

DISCUSSION

The primary goals of *CPMP* were to design, develop and evaluate a 4-year high school mathematics program that embodies the content, processes, and teaching principles recommended by the NCTM *Standards*. Evaluation data were used both to guide the development of the curriculum and to provide evidence of its impact on student learning. The *CPMP* curriculum seems to be particularly effective in developing students' conceptual understanding, quantitative thinking, and ability to solve contextualized problems. *CPMP* students perform well on tasks involving statistics and probability; content that was emphasized throughout the curriculum. At the same time, there is evidence that they are at least as well prepared for the SAT and the ACT college entrance examinations as similar students in more traditional curricula. *CPMP* students also perform well on calculus readiness tasks, which is consistent with the curriculum's focus on functions and patterns of change beginning in Course 1.

Symbolic manipulation in algebra is an area of concern for critics of *Standards*-oriented curricula. Results of the *CPMP* Posttests in Courses 1 and 2 and the NAEP-based test in Course 3 consistently show that *CPMP* students are stronger than comparison students in more traditional curricula in conceptual understanding, interpretation of algebraic representations and calculations, and problem solving in realistic contexts, but somewhat weaker in out-of-context, paper-and-pencil symbolic manipulation. This general pattern of algebraic learning is also confirmed by a study of matched groups of *CPMP* Course 3 and comparison students in Advanced Algebra in six field-test schools who were tested with a researcher-developed measure of a broad range of algebraic outcomes (Huntley, el al, 2000).

The *CPMP* development team used this evidence in the development of the field-test version of Course 4. For example, a section providing additional symbolic manipulation skill practice was included at the end of each lesson of each unit. The intent was to improve the procedural algebra outcomes while maintaining the curriculum's identified strengths. The college mathematics placement test results presented earlier suggest that this goal was achieved.

Evidence from the field tests of Courses 1–3 was also used to shape the final published version of each of these courses. As part of *CPMP*'s longitudinal study of the published curriculum mentioned at the beginning of this chapter, the Educational Testing Service's Algebra End-of-Course Examination was administered to 586 *CPMP* Course 2 students in May 1999. This test is developed, distributed, and scored by Educational Testing Service. The *CPMP* students were all the Course 2 students in the three longitudinal study schools (one suburban and two rural) in which all students have experienced a *Standards*-oriented mathematics curriculum in middle school and the *CPMP* curriculum in high school. Compared with 7,235 traditional first-time algebra students using traditional curricula who took this test in May 1999, the scores for *CPMP* students' were 9% higher in Concepts, 7% higher in Process, 10% higher in Algebraic Representations, and 6% higher in Functions. However, *CPMP* students were just 1% higher in both Algebraic Skills and Algebraic Expressions and Equations.

We believe that the *CPMP* evaluation provides strong evidence in support of the feasibility of the curriculum and of *Standards*-oriented reform generally. Nonetheless, because potentially important changes were made in the *CPMP* curriculum following its field test, new research is needed to study the effect of the "final" curriculum on student achievement outcomes in high school and post-high school settings. Ideally, such research would involve schools that have used *CPMP* for at least a few years so that teachers understand the full scope and sequence of the curriculum and that both teachers and students are accustomed to the expectations of the *CPMP* classroom environment.

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ENDNOTES

1. The research and development was funded in part by grants from the National Science Foundation (MDR-9255257 and ESI-9618193).
2. When given in Grade 9, correlation of the ITED-Q with the ITBS Mathematics total score in Grade 8 is 0.81, with students' final cumulative high school grade point average in mathematics courses is 0.59, with the ACT Mathematics test is 0.84, and with the SAT Mathematics test is 0.82. The ACT and SAT are college entrance examinations, usually completed in 11th or 12th grade.
3. Of these 1,292 students, 1,104 (85.4%) also took the ITED-Q Pretest at the beginning of Grade 9, indicating they had completed all of *CPMP* Courses 1–3. It is likely that most of the remaining 188 students transferred into *CPMP* sometime during Courses 1–3. The 1,104 students' mean score on the total NAEP-based test differed very little from that of all 1,292 students (56.1% compared with 56.4% as given in Table 14.9). Furthermore, means on the various subtests for the two groups of *CPMP* students all differed by less than 1 percentage point.