

Scope & Sequence

algebra and functions

statistics and probability

geometry and trigonometry

discrete mathematics

Contemporary Mathematics in Context

A Unified Approach

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Glencoe/McGraw-Hill

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This project was supported, in part, by the National Science Foundation.
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Send all inquiries to:
Glencoe McGraw-Hill
8787 Orion Place
Columbus, Ohio 43240-4027

ISBN 1-57039-895-X

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2 3 4 5 6 7 8 9 079 03 02 01

About the *Core-Plus Mathematics Project*

The **Core-Plus Mathematics Project (CPMP)** was funded by the National Science Foundation to develop student and teacher materials for a comprehensive Standards-based high school mathematics curriculum. Courses 1–3 comprise a core curriculum appropriate for *all* students. Course 4 continues the preparation of students for college mathematics.

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UNIFIED MATHEMATICS

Contemporary Mathematics in Context is a four-year unified curriculum that replaces the traditional Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence. Each course features interwoven strands of algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. Each of these strands is developed within focused units connected by fundamental ideas such as symmetry, functions, matrices, and data analysis and curve-fitting. The largest number of units is devoted to algebra and functions. By encountering mathematics every year from a more mathematically sophisticated point of view, students' understanding of the strands deepen across the four-year curriculum. Mathematical connections between strands and ways of thinking mathematically that are common across strands are emphasized. These mathematical habits of mind include visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, and providing convincing arguments and proofs.

Algebra and Functions

The algebra and functions strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, polynomial, logarithmic, rational, and periodic functions. Each algebraic model is investigated in four linked representations—verbal, graphic, numeric, and symbolic—with the aid of technology. Attention is also given to modeling with systems, both linear and nonlinear, and to symbolic reasoning and manipulation.

Statistics and Probability

The primary role of the statistics and probability strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. The ultimate goal is for students to understand how inferences can be made about a population by looking at a sample from that population. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

Geometry and Trigonometry

The primary goal of the geometry and trigonometry strand is to develop visual thinking and student ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. Activities include describing patterns with regard to shape, size, and location; representing patterns with drawings, coordinates, or vectors; predicting changes and invariants in shapes; and organizing geometric facts and relationships through deductive reasoning.

Discrete Mathematics

The discrete mathematics strand develops student ability to model and solve problems involving enumeration, sequential change, decision making in finite settings, and relationships among a finite number of elements. Topics include matrices, vertex-edge graphs, recursion, models of social decision making, and systematic counting methods. Key themes are discrete mathematical modeling, existence (*Is there a solution?*), optimization (*What is the best solution?*), and algorithmic problem solving (*Can you efficiently construct a solution?*).

ORGANIZATION OF THE CURRICULUM

The first three courses in the *Contemporary Mathematics in Context* series provided a common core of broadly useful mathematics for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. Course 4 formalizes and extends the core program, with a focus on the mathematics needed to be successful in college mathematics and statistics courses. Unit titles for the four-year curriculum are given in the following table.

Course 1

- 1 Patterns in Data
- 2 Patterns of Change
- 3 Linear Models
- 4 Graph Models
- 5 Patterns in Space and Visualization
- 6 Exponential Models
- 7 Simulation Models
- CAPSTONE Planning a Benefits Carnival

Course 2

- 1 Matrix Models
- 2 Patterns of Location, Shape, and Size
- 3 Patterns of Association
- 4 Power Models
- 5 Network Optimization
- 6 Geometric Form and Its Function
- 7 Patterns in Chance
- CAPSTONE Forests, the Environment, and Mathematics

Course 3

- 1 Multiple-Variable Models
- 2 Modeling Public Opinion
- 3 Symbol Sense and Algebraic Reasoning
- 4 Shapes and Geometric Reasoning
- 5 Patterns in Variation
- 6 Families of Functions
- 7 Discrete Models of Change
- CAPSTONE Making the Best of It: Optimal Forms and Strategies

Course 4

- 1 Rates of Change
- 2 Modeling Motion
- 3 Logarithmic Functions and Data Models
- 4 Counting Models
- 5 Binomial Distributions and Statistical Inference
- 6 Polynomial and Rational Functions
- 7 Functions and Symbolic Reasoning
- 8 Space Geometry
- 9 Informatics
- 10 Problem Solving, Algorithms, and Spreadsheets

The Strand Charts

The following charts provide an overview of the mathematical content and flow of Courses 1–4 in the *Contemporary Mathematics in Context* curriculum. The charts are organized by mathematical strand: algebra and functions, statistics and probability, geometry and trigonometry, and discrete mathematics. Each of the four strands has been divided into major categories, and under each of these categories you will find the mathematical topics developed in the curriculum.

Many cells in the grid have either a “●” or a “+” to indicate the units in which each topic is covered. The “●” indicates focus; this means that the topic is initially developed or is extended beyond its initial development or use. The “+” indicates connection, which means that a conceptual basis for the topic is developed, the topic is briefly introduced, or the topic is revisited and used without further development.

ALGEBRA AND FUNCTIONS

● Focus + Connections

	Course 1							Course 2							Course 3							Course 4									
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	8	9	10
Algebraic Representations																															
Coordinate graphs	●	●	+				+																								
Tables and graphs	+	●	●	+			●																								
Symbolic expressions	+	●	●	●	●	●	●																								
Modeling data patterns																															
Error of prediction and residuals																															
Matrices																															
Coordinate models of transformations																															
Summation notation																															
Vectors																															
Parametric equations																															
Logarithmic scales																															
Standard form of complex numbers																															
Polar coordinates																															
Polar form of complex numbers																															
Trigonometric form of complex numbers																															
Spreadsheets																															

Linear Expressions and Relations

Modeling situations																													
Symbolic forms and effects of parameters																													
Graphs and equations of lines																													
Solving equations																													
Solving inequalities																													
Rates of change, slopes, and intercepts																													
Parametric equations for linear motion																													

Exponential Expressions and Relations

Modeling situations																													
Symbolic forms and effects of parameters																													
Graphs and intercepts																													
Solving equations and inequalities																													
Rates of change																													
Asymptotes																													

ALGEBRA AND FUNCTIONS

● Focus	+ Connections	Course 1	Course 2	Course 3	Course 4
Power Expressions and Relations					
Symbolic forms and effects of parameters		+			+
Graphs, intercepts, and zeroes		+			+
Fractional and negative exponents					+
Laws of exponents					+
Modeling situations					+
Inverse variation					●
Asymptotes (inverse variation)					+
Solving equations and inequalities					
Roots and radicals					+
Rates of change					+

Quadratic Expressions and Relations					
Modeling situations		+			
Symbolic forms and effects of parameters		+			
Graphs, intercepts, and zeroes		+			
Rates of change		+			
Solving equations by graphic and numeric approximation		+			
Number of solutions					
Solving inequalities					
Higher-degree polynomials					
Solving equations by factoring					
Solving equations by using the quadratic formula					
Parametric equations for projectile motion					
Parametric equations for circular motion					
Parametric equations for elliptical motion					
Conic sections					

Polynomial Expressions and Relations					
Modeling situations					
Symbolic forms and effects of parameters					
Graphs, intercepts, and zeroes					
Rates of change					
End behavior					
Solving equations and inequalities					
Number of solutions					

STATISTICS AND PROBABILITY

● Focus + Connections

	Course 1							Course 2							Course 3							Course 4									
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	8	9	10
Probability																															
Normal distribution																															
Simulation																															
Random digit tables and random number generators																															
Law of Large Numbers																															
Empirical (experimental) and theoretical probabilities																															
Independent events																															
Binomial distribution																															
Geometric (waiting-time) distribution																															
Multiplication Rule for independent events																															
Geometric (area) models																															
Conditional probability																															
Expected value																															
Rare events																															
Sampling distributions																															
Mutually exclusive events																															
Addition Rule for mutually exclusive events																															
Central Limit Theorem																															
Probability distribution																															
General multiplication rule																															
Binomial probability formula																															
Normal approximation to a binomial																															
Inferential Statistics																															
Population vs. sample																															
Confidence intervals																															
Margin of error																															
Control (run) charts																															
Tests of significance including Type I and Type II errors																															
Statistical significance																															
One sample z-test for a proportion																															
Generalizability of results																															
Surveys																															
Characteristics of a well-designed survey																															
Sample survey vs. census																															
Sample size																															
Simple random sample																															
Bias in survey method																															
Response rate																															

STATISTICS AND PROBABILITY

	Course 1							Course 2							Course 3							Course 4									
● Focus	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	8	9	10
+ Connections																															
Experiments																															
Characteristics of a well-designed experiment																															
Treatments, control groups, random assignment, and replication																															
Source of bias and confounding, including placebo effect and blinding																															
Randomization test																															
Large sample test for a difference between two proportions																															

GEOMETRY AND TRIGONOMETRY

● Focus + Connections

	Course 1							Course 2							Course 3							Course 4									
	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	1	2	3	4	5	6	7	8	9	10
Trigonometry																															
Periodic change																															
Triangular linkages with one variable side																															
Trigonometric ratios and functions (sine, cosine, and tangent)																															
Indirect measurement (angles and lengths)																															
Circular and periodic motion																															
Period																															
Periodic models																															
Trigonometric graphs																															
Radian and degree measure																															
Amplitude																															
Law of Cosines																															
Law of Sines																															
Component analysis of vectors																															
Inverse trigonometric functions																															
Trigonometric identities																															
Trigonometric ratios and functions (secant, cosecant, and cotangent)																															
Reasoning and Proof																															
Inductive reasoning																															
Counterexamples																															
Deductive reasoning																															
Designing and programming algorithms																															
Coordinate and analytic proof																															
Synthetic proof																															
Assumptions in proof																															
Principles of logic																															
Converse																															
Angle relations for two intersecting lines																															
Angle relations for three intersecting lines																															
Parallelism																															
Similarity conditions for triangles																															
Congruence conditions for triangles																															
Necessary and sufficient conditions for parallelograms																															
Geometric constructions																															
Vector proof																															

DISCRETE MATHEMATICS

● Focus + Connections

Course 4
1 2 3 4 5 6 7 8 9 10

Course 3
1 2 3 4 5 6 7

Course 2
1 2 3 4 5 6 7

Course 1
1 2 3 4 5 6 7

Social Decision Making

Preferential voting																				
Approval voting	●																			
Vote-analysis methods	●																			
Arrow's Theorem	●																			

Algorithms

Optimization																				
Algorithmic problem solving																				
Designing and programming algorithms																				
Analyzing and comparing algorithms																				

Combinatorics

Counting trees																				
Multiplication principle of counting																				
Addition principle of counting																				
Pigeonhole principle																				
Permutations																				
Combinations																				
Selections with repetition																				
Pascal's triangle																				
Binomial Theorem																				
Combinatorial reasoning																				

Informatics

Set operations																				
Logical (or Boolean) operators																				
Venn diagrams																				
Modular arithmetic																				
Symmetric-key cryptosystems																				
RSA public-key cryptosystems																				
Protocols																				
Error-detecting codes																				

