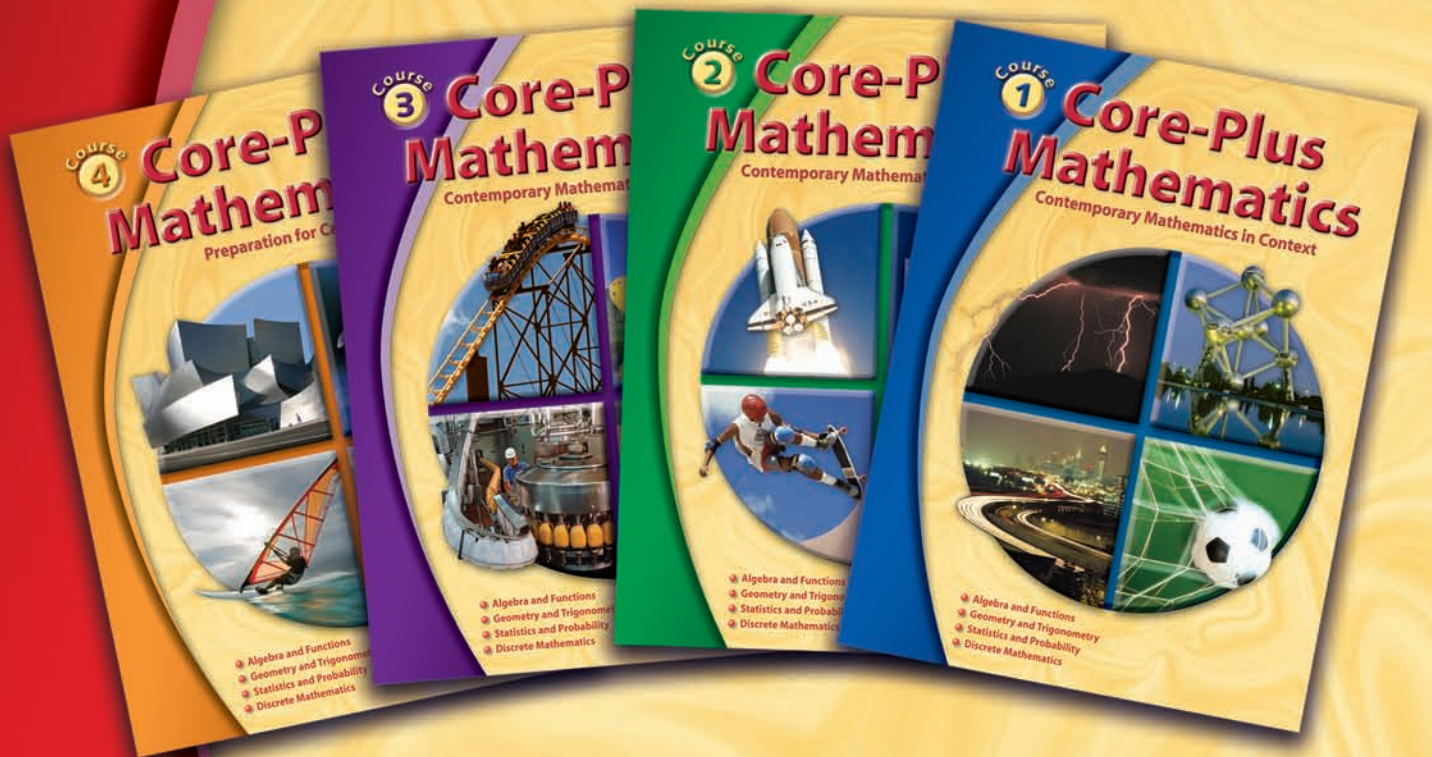


# Core-Plus Mathematics

Contemporary Mathematics in Context



## Scope and Sequence

- Algebra and Functions
- Geometry and Trigonometry
- Statistics and Probability
- Discrete Mathematics

# Core-Plus Mathematics

Contemporary Mathematics in Context

## Scope and Sequence

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*with*

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**Glencoe**



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**Core-Plus Mathematics**  
*Contemporary Mathematics in Context*  
Scope and Sequence

# About the Core-Plus Mathematics Project

The **Core-Plus Mathematics Project (CPMP)** was funded by the National Science Foundation to develop student and teacher materials for a comprehensive Standards-based high school mathematics curriculum. Courses 1–3 comprise a core program appropriate for *all* students. Course 4 continues the preparation of students for college mathematics and statistics.

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# Core-Plus Mathematics 2

## Field-Test Sites

*Core-Plus Mathematics 2* builds on the strengths of the 1st edition which was shaped by multi-year field tests in 36 high schools in Alaska, California, Colorado, Georgia, Idaho, Iowa, Kentucky, Michigan, Ohio, South Carolina, and Texas. Each revised text is the product of a three-year cycle of research and development, pilot testing and refinement, and field testing and further refinement. Special thanks are extended to the following teachers and their students who participated in the testing and evaluation of the 2nd Edition materials.

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# A Balanced and Unified Curriculum

*Core-Plus Mathematics* is a four-year unified curriculum that replaces the Algebra-Geometry-Advanced Algebra/Trigonometry-Precalculus sequence. Each course features interwoven strands of algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of these strands is developed within coherent focused units connected by fundamental ideas such as symmetry, functions, matrices, and data analysis and curve-fitting. By actively investigating mathematics and its applications every year from an increasingly more mathematically sophisticated point of view, students' understanding of the mathematics in each strand deepens across the four-year curriculum. Mathematical connections between strands and ways of thinking mathematically that are common across strands are emphasized. These mathematical habits of mind include visual thinking, recursive thinking, searching for and explaining patterns, making and checking conjectures, reasoning with multiple representations, and providing convincing arguments and proofs.

## **Algebra and Functions**

The Algebra and Functions strand develops student ability to recognize, represent, and solve problems involving relations among quantitative variables. Central to the development is the use of functions as mathematical models. The key algebraic models in the curriculum are linear, exponential, power, polynomial, logarithmic, rational, and trigonometric functions. Modeling with systems of equations, both linear and nonlinear, is developed. Attention is also given to symbolic reasoning and manipulation.

## **Geometry and Trigonometry**

The primary goal of the Geometry and Trigonometry strand is to develop visual thinking and ability to construct, reason with, interpret, and apply mathematical models of patterns in visual and physical contexts. The focus is on describing patterns in shape, size, and location; representing patterns with drawings, coordinates, or vectors; predicting changes and invariants in shapes under transformations; and organizing geometric facts and relationships through deductive reasoning.

## **Statistics and Probability**

The primary role of the Statistics and Probability strand is to develop student ability to analyze data intelligently, to recognize and measure variation, and to understand the patterns that underlie probabilistic situations. The ultimate goal is for students to understand how inferences can be made about a population by looking at a random sample from that population. Graphical methods of data analysis, simulations, sampling, and experience with the collection and interpretation of real data are featured.

## **Discrete Mathematics**

The Discrete Mathematics strand develops student ability to solve problems using vertex-edge graphs, recursion, matrices, systematic counting methods (combinatorics), and mathematical methods for democratic decision making and information processing. Key themes are discrete mathematical modeling, optimization, and algorithmic problem solving.

# Organization of the Curriculum

The first three courses in the *Core-Plus Mathematics* series provide a significant core of broadly useful mathematics for all students. They were developed to prepare students for success in college, in careers, and in daily life in contemporary society. Course 4: Preparation for Calculus formalizes and extends the core program, with a focus on the mathematics needed to be successful in undergraduate programs requiring calculus. Unit titles for the four courses are given in the following table. Focus and content of these units are described on pages 3–7.

## Course 1

- 1 Patterns of Change
- 2 Patterns in Data
- 3 Linear Functions
- 4 Vertex-Edge Graphs
- 5 Exponential Functions
- 6 Patterns in Shape
- 7 Quadratic Functions
- 8 Patterns in Chance

## Course 2

- 1 Functions, Equations, and Systems
- 2 Matrix Methods
- 3 Coordinate Methods
- 4 Regression and Correlation
- 5 Nonlinear Functions and Equations
- 6 Network Optimization
- 7 Trigonometric Methods
- 8 Probability Distributions

## Course 3

- 1 Reasoning and Proof
- 2 Inequalities and Linear Programming
- 3 Similarity and Congruence
- 4 Samples and Variation
- 5 Polynomial and Rational Functions
- 6 Circles and Circular Functions
- 7 Recursion and Iteration
- 8 Inverse Functions

## Course 4: Preparation for Calculus

- 1 Families of Functions
- 2 Vectors and Motion
- 3 Algebraic Functions and Equations
- 4 Trigonometric Functions and Equations
- 5 Exponential Functions, Logarithms, and Data Modeling
- 6 Surfaces and Cross Sections
- 7 Concepts of Calculus
- 8 Counting Methods and Induction

# Core-Plus Mathematics 2nd Edition

## Course 1 Units

**Unit 1** **Patterns of Change** develops student ability to recognize and describe important patterns that relate quantitative variables, to use data tables, graphs, words, and symbols to represent the relationships, and to use reasoning and calculating tools to answer questions and solve problems.

*Topics include* variables and functions, algebraic expressions and recurrence relations, coordinate graphs, data tables and spreadsheets, and equations and inequalities.

**Unit 2** **Patterns in Data** develops student ability to make sense of real-world data through use of graphical displays, measures of center, and measures of variability.

*Topics include* distributions of data and their shapes, as displayed in dot plots, histograms, and box plots; measures of center including mean and median, and their properties; measures of variability including interquartile range and standard deviation, and their properties; and percentiles and outliers.

**Unit 3** **Linear Functions** develops student ability to recognize and represent linear relationships between variables and to use tables, graphs, and algebraic expressions for linear functions to solve problems in situations that involve constant rate of change or slope.

*Topics include* linear functions, slope of a line, rate of change, modeling linear data patterns, solving linear equations and inequalities, equivalent linear expressions.

**Unit 4** **Vertex-Edge Graphs** develops student understanding of vertex-edge graphs and ability to use these graphs to represent and solve problems involving paths, networks, and relationships among a finite number of elements, including finding efficient routes and avoiding conflicts.

*Topics include* vertex-edge graphs, mathematical modeling, optimization, algorithmic problem solving, Euler circuits and paths, matrix representation of graphs, vertex coloring and chromatic number.

**Unit 5** **Exponential Functions** develops student ability to recognize and represent exponential growth and decay patterns, to express those patterns in symbolic forms, to solve problems that involve exponential change, and to use properties of exponents to write expressions in equivalent forms.

*Topics include* exponential growth and decay functions, data modeling, growth and decay rates, half-life and doubling time, compound interest, and properties of exponents.

**Unit 6** **Patterns in Shape** develops student ability to visualize and describe two- and three-dimensional shapes, to represent them with drawings, to examine shape properties through both experimentation and careful reasoning, and to use those properties to solve problems.

*Topics include* Triangle Inequality, congruence conditions for triangles, special quadrilaterals and quadrilateral linkages, Pythagorean Theorem, properties of polygons, tilings of the plane, properties of polyhedra, and the Platonic solids.

**Unit 7** **Quadratic Functions** develops student ability to recognize and represent quadratic relations between variables using data tables, graphs, and symbolic formulas, to solve problems involving quadratic functions, and to express quadratic polynomials in equivalent factored and expanded forms.

*Topics include* quadratic functions and their graphs, applications to projectile motion and economic problems, expanding and factoring quadratic expressions, and solving quadratic equations by the quadratic formula and calculator approximation.

**Unit 8** **Patterns in Chance** develops student ability to solve problems involving chance by constructing sample spaces of equally-likely outcomes or geometric models and to approximate solutions to more complex probability problems by using simulation.

*Topics include* sample spaces, equally-likely outcomes, probability distributions, mutually exclusive (disjoint) events, Addition Rule, simulation, random digits, discrete and continuous random variables, Law of Large Numbers, and geometric probability.

## Course 2 Units

**Unit 1** **Functions, Equations, and Systems** reviews and extends student ability to recognize, describe, and use functional relationships among quantitative variables, with special emphasis on relationships that involve two or more independent variables.

*Topics include* direct and inverse variation and joint variation; power functions; linear equations in standard form; and systems of two linear equations with two variables, including solution by graphing, substitution, and elimination.

**Unit 2** **Matrix Methods** develops student understanding of matrices and ability to use matrices to represent and solve problems in a variety of real-world and mathematical settings.

*Topics include* constructing and interpreting matrices, row and column sums, matrix addition, scalar multiplication, matrix multiplication, powers of matrices, inverse matrices, properties of matrices, and using matrices to solve systems of linear equations.

**Unit 3** **Coordinate Methods** develops student understanding of coordinate methods for representing and analyzing properties of geometric shapes, for describing geometric change, and for producing animations.

*Topics include* representing two-dimensional figures and modeling situations with coordinates, including computer-generated graphics; distance in the coordinate plane, midpoint of a segment, and slope; coordinate and matrix models of rigid transformations (translations, rotations, and line reflections), of size transformations, and of similarity transformations; animation effects.

**Unit 4** **Regression and Correlation** develops student understanding of the characteristics and interpretation of the least squares regression equation and of the use of correlation to measure the strength of the linear association between two variables.

*Topics include* interpreting scatterplots; least squares regression, residuals and errors in prediction, sum of squared errors, influential points; Pearson's correlation coefficient and its properties, lurking variables, and cause and effect.

**Unit 5** **Nonlinear Functions and Equations** introduces function notation, reviews and extends student ability to construct and reason with functions that model parabolic shapes and other quadratic relationships in science and economics, with special emphasis on formal symbolic reasoning methods, and introduces common logarithms and algebraic methods for solving exponential equations.

*Topics include* formalization of function concept, notation, domain and range; factoring and expanding quadratic expressions, solving quadratic equations by factoring and the quadratic formula, applications to supply and demand, break-even analysis; common logarithms and solving exponential equations using base 10 logarithms.

**Unit 6** **Network Optimization** develops student understanding of vertex-edge graphs and ability to use these graphs to solve network optimization problems.

*Topics include* optimization, mathematical modeling, algorithmic problem solving, digraphs, trees, minimum spanning trees, distance matrices, Hamilton circuits and paths, the Traveling Salesperson Problem, critical paths, and the PERT technique.

**Unit 7** **Trigonometric Methods** develops student understanding of trigonometric functions and the ability to use trigonometric methods to solve triangulation and indirect measurement problems.

*Topics include* sine, cosine, and tangent functions of measures of angles in standard position in a coordinate plane and in a right triangle; indirect measurement; analysis of variable-sided triangle mechanisms; Law of Sines and Law of Cosines.

**Unit 8** **Probability Distributions** further develops student ability to understand and visualize situations involving chance by using simulation and mathematical analysis to construct probability distributions.

*Topics include* Multiplication Rule, independent and dependent events, conditional probability, probability distributions and their graphs, waiting-time (or geometric) distributions, expected value, and rare events.

## Course 3 Units

<b>Unit 1</b>	<p><b>Reasoning and Proof</b> develops student understanding of formal reasoning in geometric, algebraic, and statistical contexts and of basic principles that underlie those reasoning strategies.</p> <p><i>Topics include</i> inductive and deductive reasoning strategies; principles of logical reasoning—Affirming the Hypothesis and Chaining Implications; relation among angles formed by two intersecting lines or by two parallel lines and a transversal; rules for transforming algebraic expressions and equations; design of experiments including the role of randomization, control groups, and blinding; sampling distribution, randomization test, and statistical significance.</p>
<b>Unit 2</b>	<p><b>Inequalities and Linear Programming</b> develops student ability to reason both algebraically and graphically to solve inequalities in one and two variables, introduces systems of inequalities in two variables, and develops a strategy for optimizing a linear function in two variables within a system of linear constraints on those variables.</p> <p><i>Topics include</i> inequalities in one and two variables (including absolute value and quadratic inequalities), number line graphs, interval notation, systems of linear inequalities, and linear programming.</p>
<b>Unit 3</b>	<p><b>Similarity and Congruence</b> extends student understanding of similarity and congruence and their ability to use those relations to solve problems and to prove geometric assertions with and without the use of coordinates.</p> <p><i>Topics include</i> connections between Law of Cosines, Law of Sines, and sufficient conditions for similarity and congruence of triangles, centers of triangles, applications of similarity and congruence in real-world contexts, necessary and sufficient conditions for parallelograms, sufficient conditions for congruence of parallelograms, and midpoint connector theorems.</p>
<b>Unit 4</b>	<p><b>Samples and Variation</b> extends student understanding of the measurement of variability, develops student ability to use the normal distribution as a model of variation, introduces students to the binomial distribution and its use in decision making, and introduces students to the probability and statistical inference involved in control charts used in industry for statistical process control.</p> <p><i>Topics include</i> normal distribution, standardized scores, binomial distributions (shape, expected value, standard deviation), normal approximation to a binomial distribution, odds, statistical process control, control charts, and the Central Limit Theorem.</p>
<b>Unit 5</b>	<p><b>Polynomial and Rational Functions</b> extends student ability to represent and draw inferences about polynomial and rational functions using symbolic expressions and manipulations.</p> <p><i>Topics include</i> definition and properties of polynomials, operations on polynomials; completing the square, proof of the quadratic formula, solving quadratic equations (including complex number solutions), vertex form of quadratic functions; definition and properties of rational functions, operations on rational expressions.</p>
<b>Unit 6</b>	<p><b>Circles and Circular Functions</b> develops student understanding of relationships among special lines, segments, and angles in circles and the ability to use properties of circles to solve problems; develops student understanding of circular functions and the ability to use these functions to model periodic change; and extends student ability to reason deductively in geometric settings.</p> <p><i>Topics include</i> properties of chords, tangent lines, and central and inscribed angles of circles; linear and angular velocity; radian measure of angles; and circular functions as models of periodic change.</p>
<b>Unit 7</b>	<p><b>Recursion and Iteration</b> extends student ability to represent, analyze, and solve problems in situations involving sequential and recursive change.</p> <p><i>Topics include</i> iteration and recursion as tools to model and analyze sequential change in real-world contexts, including compound interest and population growth; arithmetic, geometric, and other sequences; arithmetic and geometric series; finite differences; linear and nonlinear recurrence relations; and function iteration, including graphical iteration and fixed points.</p>
<b>Unit 8</b>	<p><b>Inverse Functions</b> develops student understanding of inverses of functions with a focus on logarithmic functions and their use in modeling and analyzing problem situations and data patterns.</p> <p><i>Topics include</i> inverses of functions; logarithmic functions and their relation to exponential functions, properties of logarithms, equation solving with logarithms; and inverse trigonometric functions and their applications to solving trigonometric equations.</p>

## Course 4 Units: Preparation for Calculus

**Unit 1** **Families of Functions** extends student understanding of linear, exponential, quadratic, power, and trigonometric functions to model data patterns whose graphs are transformations of basic patterns; and develops understanding of operations on functions useful in representing and reasoning about quantitative relationships.

*Topics include* linear, exponential, quadratic, power, and trigonometric functions; data modeling; translation, reflection, and stretching of graphs; and addition, subtraction, multiplication, division, and composition of functions.

**Unit 2** **Vectors and Motion** develops student understanding of two-dimensional vectors and their use in modeling linear, circular, and other nonlinear motion.

*Topics include* concept of vector as a mathematical object used to model situations defined by magnitude and direction; equality of vectors, scalar multiples, opposite vectors, sum and difference vectors, dot product of two vectors, position vectors and coordinates; and parametric equations for motion along a line and for motion of projectiles and objects in circular and elliptical orbits.

**Unit 3** **Algebraic Functions and Equations** reviews and extends student understanding of properties of polynomial and rational functions and skills in manipulating algebraic expressions and solving polynomial and rational equations, and develops student understanding of complex number representations and operations.

*Topics include* polynomials, polynomial division, factor and remainder theorems, operations on complex numbers, representation of complex numbers as vectors, solution of polynomial equations, rational function graphs and asymptotes, and solution of rational equations and equations involving radical expressions.

**Unit 4** **Trigonometric Functions and Equations** extends student understanding of, and ability to reason with, trigonometric functions to prove or disprove two trigonometric expressions are identical and to solve trigonometric equations; to geometrically represent complex numbers and complex number operations and to find roots of complex numbers.

*Topics include* the tangent, cotangent, secant, and cosecant functions; fundamental trigonometric identities, sum and difference identities, double-angle identities; solving trigonometric equations and expression of periodic solutions; rectangular and polar representations of complex numbers, absolute value, DeMoivre's Theorem, and the roots of complex numbers.

**Unit 5** **Exponential Functions, Logarithms, and Data Modeling** extends student understanding of exponential and logarithmic functions to the case of natural exponential and logarithmic functions, solution of exponential growth and decay problems, and use of logarithms for linearization and modeling of data patterns.

*Topics include* exponential functions with rules in the form  $f(x) = Ae^{kx}$ , natural logarithm function, linearizing bivariate data and fitting models using log and log-log transformations.

**Unit 6** **Surfaces and Cross Sections** extends student ability to visualize and represent three-dimensional shapes using contours, cross sections, and reliefs, and to visualize and represent surfaces and conic sections defined by algebraic equations.

*Topics include* using contours to represent three-dimensional surfaces and developing contour maps from data; sketching surfaces from sets of cross sections; conics as planar sections of right circular cones and as locus of points in a plane; three-dimensional rectangular coordinate system; sketching surfaces using traces, intercepts and cross sections derived from algebraically-defined surfaces; and surfaces of revolution and cylindrical surfaces.

**Unit 7** **Concepts of Calculus** develops student understanding of fundamental calculus ideas through explorations in a variety of applied problem contexts and their representations in function tables and graphs.

*Topics include* instantaneous rates of change, linear approximation, area under a curve, and applications to problems in physics, business, and other disciplines.

**Unit 8** : **Counting Methods and Induction** extends student ability to count systematically and solve enumeration problems, and develops understanding of, and ability to do, proof by mathematical induction.

: *Topics include* systematic listing and counting, counting trees, the Multiplication Principle of Counting, Addition Principle of Counting, combinations, permutations, selections with repetition; the binomial theorem, Pascal's triangle, combinatorial reasoning; the general multiplication rule for probability; the Principle of Mathematical Induction; and the Least Number Principle.

## **Mathematics Content Elaborated**

### **Strand Charts**

The mathematical content of the four courses of *Core-Plus Mathematics* are elaborated in the remainder of this booklet. Pages 9–21 provide strand charts and pages 22–26 provide an alignment with the Common Core State Standards Mathematical Practices.

The following charts provide an overview of the mathematical content and flow of Courses 1–4 in the *Core-Plus Mathematics* curriculum. The charts are organized by mathematical strand: algebra and functions, geometry and trigonometry, statistics and probability, and discrete mathematics. Each of the four strands has been divided into major content categories, and under each of these categories you will find the key mathematical topics developed in the curriculum.

Many cells in the grid have either a “F” or a “C” to indicate the units in which each topic is treated. The “F” indicates *focus*; this means that the topic is initially developed or is extended beyond its initial development or use. The “C” indicates *connections*, which means that a conceptual basis for the topic is developed, the topic is informally introduced, or the topic is revisited and used without further development.

To help build and maintain proficiency with key topics in the charts, Review tasks in each lesson of each unit provide students distributed practice with related concepts and skills. These practice opportunities are not referenced in the following strand charts.



# ALGEBRA AND FUNCTIONS

	Course 1								Course 2								Course 3								Course 4							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
<b>F : Focus</b>																																
<b>C : Connections</b>																																
<b>Power Expressions and Relations</b>																																
Symbolic forms and effects of parameters	C							F				C									C											C
Graphs, intercepts, and zeroes	C							F				C									C											C
Fractional and negative exponents								F																								
Laws of exponents								F																								
Modeling situations																																
Inverse variation																																
Asymptotes (inverse variation)																																
Solving equations and inequalities																																
Roots and radicals																																
Rates of change	F							C																								F

	Course 1								Course 2								Course 3								Course 4							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
<b>Quadratic Expressions and Relations</b>																																
Modeling situations	C	C						F																								F
Symbolic forms and effects of parameters	C	C						F																								C
Graphs, intercepts, and zeroes	C							F																								C
Rates of change	C							C																								
Solving equations and inequalities by graphic and numeric approximation	C							F																								
Number of solutions								F																								
Solving inequalities								F																								
Solving equations by factoring								C																								
Solving equations by using the quadratic formula								F																								
Parametric equations for projectile motion																																
Parametric equations for circular motion																																
Parametric equations for elliptical motion																																
Conic sections								C																								F

	Course 1								Course 2								Course 3								Course 4							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
<b>Polynomial Expressions and Relations</b>																																
Modeling situations																																
Symbolic forms and effects of parameters																																C
Graphs, intercepts, and zeroes																																
Rates of change																																
End behavior																																
Solving equations and inequalities																																
Number of solutions																																

# ALGEBRA AND FUNCTIONS

F : Focus	Course 1								Course 2								Course 3								Course 4							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
Proportions																																
Symbolic forms and effects of parameters								C																								
Graphs, intercepts, and zeroes								C																								
Rates of change																																
Modeling situations																																
Solving equations and inequalities																																
Asymptotes																																
Simplifying: adding, subtracting, multiplying, and																																

## Rational Expressions and Relations

Proportions																																
Symbolic forms and effects of parameters																																
Graphs, intercepts, and zeroes																																
Rates of change																																
Modeling situations																																
Solving equations and inequalities																																
Asymptotes																																
Simplifying: adding, subtracting, multiplying, and																																

## Periodic Relations

Modeling situations																																
Symbolic forms and effects of parameters																																
Graphs, intercepts, and zeroes																																
Rates of change																																
Inverse trigonometric functions																																
Solving trigonometric equations																																
Trigonometric identities																																

## Logarithmic Expressions and Relations

Definition and notation																																
Common logarithms																																
Modeling situations																																
Graphs and intercepts																																
Symbolic forms and effects of parameters																																
Properties of logarithms																																
Solving logarithmic equations and inequalities																																
Evaluating logarithmic expressions																																
Natural logarithms																																
Change of base for logarithms																																





# GEOMETRY AND TRIGONOMETRY

F : Focus    C : Connections

Course 1    Course 2    Course 3    Course 4

## Two-dimensional Figures

	1	2	3	4	5	6	7	8
Angle relations for two intersecting lines								
Parallel lines and angle relations								
Triangles and their properties				F				
Quadrilaterals and their properties				C				
Parallelograms and their properties				F				
Triangular and quadrilateral linkages				F				
Polygons and their properties				C				
Geometric constructions								
Circles and their properties				F				
Conic sections and their properties								
Bilateral and rotational symmetry				C				
Tessellations								
Translational symmetry								
Fractals								
Congruence								
Congruence conditions for triangles				F				
Similarity				C				
Similarity conditions for triangles								

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

## Three-dimensional Figures

Right polygonal prisms and pyramids								
Cones, cylinders, and spheres								
Sketching shapes								
Rigidity								
Bilateral and rotational symmetry								
Regular (Platonic) solids				C				
Cross sections								
Contour diagrams								
Surfaces								
Surfaces of revolution								
Cylindrical surfaces								

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8









# STATISTICS AND PROBABILITY

F : Focus      C : Connections

Course 1      Course 2      Course 3      Course 4

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

1 | 2 | 3 | 4 | 5 | 6 | 7 | 8

## Surveys Statistical Studies

Sample survey									
Experiment									
Sample size									
Simple random sample									
Observational study									


F									
F									
C									
C									
F									


## Experiments

Characteristics of a well-designed experiments: treatments, control groups, random assignment, and replication									
Source of bias and confounding, including placebo effect and blinding									
Randomization test									


F									
F									
F									


# DISCRETE MATHEMATICS

	Course 1								Course 2								Course 3								Course 4							
	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8
<b>F</b> : Focus																																
<b>C</b> : Connections																																
<b>Vertex-Edge Graphs</b>																																
Vertex-edge graph models																																
Digraphs																																
Adjacency matrices for vertex-edge graphs																																
Euler paths and circuits																																
Graph coloring																																
Critical path analysis and PERT charts																																
Trees and minimum spanning trees																																
Hamilton paths and circuits (including TSP)																																

## Recursion and Iteration

Informal representation with words like <i>NOW</i> and <i>NEXT</i>																																
Recursive representation of linear functions																																
Recursive representation of exponential functions																																
Recursive representation of polynomial functions																																
Recurrence relations																																
Sequences and series																																
Finite differences																																
Function iteration																																
Fixed points																																
Graphical iteration																																
Fractals																																

## Matrices

Matrix models																																
Row and column sums																																
Matrix addition																																
Scalar multiplication																																
Matrix multiplication																																
Identity matrices																																
Inverse matrices																																
Properties of matrices																																
Matrix solutions of linear systems																																
Adjacency matrices for vertex-edge graphs																																
Transformation matrices																																
Scatterplot matrices																																



# Common Core State Standards for Mathematics Correlated to Core-Plus Mathematics: Course 1, Course 2, Course 3, and Course 4: Preparation for Calculus

The *Core-Plus Mathematics* curriculum, by design, incorporates the Common Core State Standards (CCSS) Mathematical Practices into each lesson. Descriptions of the Mathematical Practices along with selected examples of these practices in each Core-Plus Mathematics text are provided in the following table. A complete correlation that includes the content standards is available from your Glencoe sales representative or at [www.wmich.edu/cpmp](http://www.wmich.edu/cpmp).

Standards for Mathematical Practice	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<p><b>1. Make sense of problems and persevere in solving them.</b></p> <p>Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.</p>	<p>Throughout Units 1–8</p> <p>Examples: 8–10, 56–58, 61 #11, 93 #8, 111, 131, 159, 239–242, 297 STM, 413 #3, 420–421</p>	<p>Throughout Units 1–8</p> <p>Examples: 61–64, 121 #7, 124 #11, 141 Check Your Understanding (CYU) 280–285, 314 #13, 364–367, 372 #17, 474 #2, #3</p>	<p>Throughout Units 1–8</p> <p>Examples: 32–33, 149 #14, 233 #7, 360 #15, 432 CYU, 435 #7, 440 #8, 450 #33, 459–461, 470 #4, 554 #22</p>	<p>Throughout Units 1–8</p> <p>Examples: 71 #22, 178–180, 231 CYU, 288 #7, 300–301, 318–321, 394–398, 470–473, 496–501, 591–595, 611–614</p>

# Common Core State Standards for Mathematics Correlated to Core-Plus Mathematics: Course 1, Course 2, Course 3, and Course 4: Preparation for Calculus

Standards for Mathematical Practice	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<p><b>2. Reason abstractly and quantitatively.</b></p> <p>Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to <i>decontextualize</i>—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to <i>contextualize</i>, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.</p>	<p>Throughout Units 1–3, 5, 8</p> <p>Examples: 27–31, 116–123, 194–197, 250–254, 307–311, 369 CYU, 469–472</p>	<p>Throughout Units 1–8</p> <p>Examples: 70–71, 104–117, 145 #2, 223 #15, 331, 349–350 #17, 472 #4, 489–497, 552–553</p>	<p>Throughout Units 1–8</p> <p>Examples: 118 #1, #2, 145–150, 220 #18, 338–339, 353 #1, 562 CYU, 468–471</p>	<p>Throughout Units 1–8</p> <p>Examples: 68 #10, 146 #3, 157–161, 239–242, 305–306, 364–367, 399–404, 439–442, 490–495</p>
<p><b>3. Construct viable arguments and critique the reasoning of others.</b></p> <p>Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.</p>	<p>Throughout Units 1–7</p> <p>Examples: 180 #30, 194 #4, 335–337, 348 #29, 374–377, 387, 493 #2, 504 #13, 517 CYUb</p>	<p>Throughout Units 1–7</p> <p>Examples: 22 #14, 54–55, 111 #7, 123 #10, 168 #9, 170–180, 179 #8, 207 #3, #4, 490–491</p>	<p>Throughout Units 1–8</p> <p>Examples: 2–15, 166 #5, 171 #5, 204–208, 233 #7, 352 #9, 405–406, 487 #11, 566–567</p>	<p>Throughout Units 1–8</p> <p>Examples: 124 #19, 287–294, 315 #5, 316 #10, 326 #22, 434 #6, 472 #8, 594–595, 598 #8, 604–614</p>

# Common Core State Standards for Mathematics Correlated to Core-Plus Mathematics: Course 1, Course 2, Course 3, and Course 4: Preparation for Calculus

Standards for Mathematical Practice	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<p><b>4. Model with mathematics.</b></p> <p>Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.</p>	<p>Throughout Units 1–8</p> <p>Examples: 161–167, 280 #10, 323–332, 363–369, 383, 463–468, 551–561</p>	<p>Throughout Units 1–8</p> <p>Examples: 1 58–160, 232–242, 260–268, 439–442, 518–519, 522–531, 569 CYU, 573–576</p>	<p>Throughout Units 1–8</p> <p>Examples: 81–88, 132–143, 218 #12, 231 #5, 242–247, 360 #14, 435–437, 495–506</p>	<p>Throughout Units 1–8</p> <p>Examples: 126 #23, 157–161, 182–187, 425–429, 436 #9, 464–470, 482 #26, 501, 512–516</p>
<p><b>5. Use appropriate tools strategically.</b></p> <p>Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.</p>	<p>Throughout Units 1–8</p> <p>Examples: 52–56, 137 #16, 161–167, 281 #13, 297, 480–482, 568–570</p>	<p>Throughout Units 1–8</p> <p>Examples: 49–57, 97 #18, 280–298, 385, 405 #12, 498–501, 507 #11, 581</p>	<p>Throughout Units 1–8</p> <p>Examples: 93–95, 220 #19, 254 #13, 260–265, 462–467, 475 #11, 519–522</p>	<p>Throughout Units 1–8</p> <p>Examples: 138–144, 451 #31, 468–470, 483 #28, 505–507, 518 #20, 536–537, 591–595</p>

# Common Core State Standards for Mathematics Correlated to Core-Plus Mathematics: Course 1, Course 2, Course 3, and Course 4: Preparation for Calculus

Standards for Mathematical Practice	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<p><b>6. Attend to precision.</b></p> <p>Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.</p>	<p>Throughout Units 1–8</p> <p>Examples: 20–21 #15, 175 #17, 374–377, 387 #10, 390 #17, 391 #19, 466 #5, 482 #9</p>	<p>Throughout Units 1–8</p> <p>Examples: 32–33, 92 #7, 172 #6, 261 #3, 382– 383, 467–477, 531</p>	<p>Throughout Units 1–8</p> <p>Examples: 123 #16, 165–168, 245–247, 283–296, 321–323, 581–583</p>	<p>Throughout Units 1–8</p> <p>Examples: 66 #7, 110 #4, 239 #10, 366, 493–495, 517 #15, 535 #3c, 576 #11</p>
<p><b>7. Look for and make use of structure.</b></p> <p>Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see <math>7 \times 8</math> equals the well remembered <math>7 \times 5 + 7 \times 3</math>, in preparation for learning about the distributive property. In the expression <math>x^2 + 9x + 14</math>, older students can see the <math>14</math> as <math>2 \times 7</math> and the <math>9</math> as <math>2 + 7</math>. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see <math>5 - 3(x - y)^2</math> as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers <math>x</math> and <math>y</math>.</p>	<p>Throughout Units 1–8</p> <p>Examples: 178 #24, 195, 239–247, 291–297, 303 STM, 328 STM, 468, 478</p>	<p>Throughout Units 1–8</p> <p>Examples: 26–29, 42 #21, 60 STMd, 96 CYU, 178 #6, 332–340, 351 #22, 481 #20</p>	<p>Throughout Units 1–8</p> <p>Examples: 60 #5, 62–71, 112–117, 334, 348–352, 387 #28, 489–495, 506 #19, 597 #29</p>	<p>Throughout Units 1–8</p> <p>Examples: 59–64, 141–144, 188–193, 202 #14, 250–256, 260 #14, 262 #24, 369–372, 391–393, 464–470</p>

# Common Core State Standards for Mathematics Correlated to Core-Plus Mathematics: Course 1, Course 2, Course 3, and Course 4: Preparation for Calculus

Standards for Mathematical Practice	Student Edition Lessons			
	Course 1	Course 2	Course 3	Course 4: Preparation for Calculus
<p><b>8. Look for and express regularity in repeated reasoning.</b></p> <p>Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation <math>(y - 2)/(x - 1) = 3</math>. Noticing the regularity in the way terms cancel when expanding <math>(x - 1)(x + 1)</math>, <math>(x - 1)(x^2 + x + 1)</math>, and <math>(x - 1)(x^3 + x^2 + x + 1)</math> might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.</p>	<p>Throughout Units 1–7</p> <p>Examples: 27–35, 152 #1e, 332–334, 404–406, 437–438 #6, 473–478, 497 #5</p>	<p>Throughout Units 1–7</p> <p>Examples: 23 #20, 40 #15, 210–216, 371 #13, 570–572, 578 #14</p>	<p>Throughout Units 1–8</p> <p>Examples: 62 #4, #5, 103 #3, 329–331, 341 #12, 482–489, 507, 539–542</p>	<p>Throughout Units 1–8</p> <p>Examples: 107 #7, 165–168, 205 #28, 245 #26, 318–321, 361 #7, 499 10, 501–504, 535 #3, 591–595</p>

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