Driven by applications in fields such as robotics and satellite attitude control, as well as by a need for theoretical development of appropriate tools for the analysis of geometric systems, problems of control of dynamical systems on manifolds have been studied intensively during the past three decades. In this thesis we suggest new mathematical techniques for the study of control and dynamic optimization problems on manifolds. This work has several components including: an extension of the classical Chronological Calculus to control and dynamical systems which are merely measurable in time and evolve on manifolds modeled over Banach space; novel proofs of Pontryagin Maximum Principle in settings more general than those currently existing in the literature; necessary optimality conditions for dynamic optimization problems on manifolds in which the dynamics are constrained by a differential inclusion; and a generic existence and uniqueness theorem for problems of optimal control posed on manifolds. Our studies of optimal control and dynamic optimization both include exact penalization and metric regularity results for problems with initially and terminally constrained states which are new even in the case $M = \mathbb{R}^n$. This work also includes generalizations of the classical Chow-Rashevski theorem from Geometric Control theory and the Fundamental and duBois-Reymond lemmas from classical Calculus of Variations to the setting of infinite-dimensional manifolds.