

NEW MEASURES OF WHITENESS THAT CORRELATE **BETTER WITH
PERCEIVED COLOR APPEARANCE**

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ABSTRACT

Whiteness formulas currently in use satisfactorily characterize the appearance of commercial whiteness. However, these formulas are generally unsuccessful in assessing tinted samples with chromaticities placed near the borders of “white” colors. Here, new whiteness formulas are proposed. These are compared with CIE, Hunter and Ganz whiteness formulas and TAPPI brightness, through perceptual evaluation and instrumental measurements. Both of the proposed whiteness formulas matched well with 48 randomly selected observers for printed samples. CIE, Hunter and Ganz whiteness formulas had poorer correlations with the observers’ evaluations. TAPPI brightness also agreed well with the observers’ assessments, but cannot be strictly correct because of its reliance on a narrow wavelength range. Relatively saturated chroma values are associated with the maximum whiteness values with the evaluated CIE, Hunter and Ganz whiteness formulas, but not for the proposed formulas.

1. INTRODUCTION

Over the years, many studies have been carried out and many formulas proposed for whiteness. However, disagreements and arguments have never ended.^{1,2} One important argument is about description of perfect whiteness and which directions of departure from it should be favored or avoided.¹⁻³ The arguments are further confused by the semantic difference between “white samples” and “whiteness of samples”.^{1,3} A white sample is characterized by high levels of luminosity and no saturation- therefore no hue at all.³ On the other hand, samples showing high whiteness are characterized by high levels of luminosity and finite saturation-with a blue hue.³ As such, whiteness is characterized as being contrary to yellowness.

In addition, widespread usage of fluorescent whitening agents, to improve the whiteness of objects, has compounded the disagreements in evaluating whiteness.¹⁻⁵ That is because whiteness depends on observers, and for the same observer, it also depends on the evaluation methods applied.¹⁻³ It also depends on the individual observer preference and many other varying conditions.^{2,3}

Any whiteness assessment technique is first based on the perceptual evaluation and psychometric techniques.³ Then, instrumental measurement and analyses are made and a correlation between the perceptual evaluation and instrumental measurements is searched. There have been many attempts to approximate observer preferences in regards to tint or hue of the object by assigning different weight proportions in formula parameters.⁶⁻¹⁴

2. TECHNICAL CONSIDERATIONS

A. Chromaticity Coordinates and the Chromaticity Diagram

Three color-matching functions were defined by the CIE (*Commission Internationale de l'Eclairage*) in 1931.¹⁵ These correspond to human red, green and blue color perception and are

usually called the 2° observer functions, because the CIE visual observations were conducted with a visual area subtending a 2° visual angle.¹ In 1964,¹⁶ the CIE defined an additional set of color matching functions corresponding to a 10° observer. These functions are generally used to estimate the human cone receptor's reaction to incident radiation from an object, if the radiation is known.^{1,2,6,7,17}

Spectrophotometers measure the amount of radiation versus wavelength and, therefore the cone responses can be estimated. The X, Y and Z tristimulus values are calculated in terms of the spectral response and together represent an unambiguous description of a color, calculated or measured by reference to the CIE standard illuminant and observer functions.^{5,15,16} The CIE standard observer defines three tristimulus functions $X(\lambda)$, $Y(\lambda)$, and $Z(\lambda)$. $X(\lambda)$ is equivalent to the visual efficiency function of the human eye with a peak in the red region, $Y(\lambda)$ to the green response and luminosity, and $Z(\lambda)$ to the blue region. Tristimulus colorimeters attempt to filter light to reproduce the human light cone response and X, Y, Z values can be calculated.^{5,15-17}

Each tristimulus value (X,Y,Z) is a primary of an axis in a three-dimensional space and all together a sample's tristimulus values define a position in that three-dimensional space. A two-dimensional chromaticity diagram is obtained by performing two sequential projections. Tristimulus values are converted into two variables giving a two dimensional map. That is, magnitudes of tristimulus values are transformed into ratios of tristimulus values or in other words into chromaticity coordinates,^{1,7} given by:

$$x = X/(X+Y+Z) \quad 1a$$

$$y = Y/(X+Y+Z) \quad 1b$$

$$z = Z/(X+Y+Z) \quad 1c$$

Only two of the three chromaticity coordinates or projection values are needed to describe the color in chromaticity space, since the sum of the tristimulus values equals 1. They are usually x , and y chromaticity coordinates in the CIE system. The x and y values are usually plotted on the chromaticity diagram.^{1,2} The points in this diagram are indications of the hue and saturation of the corresponding colors. One of the tristimulus values, usually the Y value, also must be specified in order to uniquely define the color. The line connecting the points representing the chromaticities of the spectrum colors, the spectrum locus, of the chromaticity diagram looks like a horseshoe as shown in Figure 1.

The CIE tristimulus system and CIE chromaticity diagram are not based on steps of equal visual perception. In other words, equal distances on the diagram do not correspond to equal visual differences, because equal chromaticity values can have arbitrary intensities. The projected values of x and y are not even perceived equally visually spaced, even for the same value of Y . This is one of the biggest disadvantages of [the](#) CIE tristimulus and chromaticity systems. However, there have been several studies conducted and proposals have been made to approach to equal perception. The CIE tristimulus system and the CIE chromaticity diagram do not intend to describe color appearance based on color perception. Rather they both intend to provide the information whether two samples match either in regards to their tristimulus values or their chromaticities. These systems do not tell what the samples look like or how the samples differ if they do not match. In reality, a given chromaticity can have a wide variety of appearances depending on the viewing conditions, and illumination.¹

As explained and shown above, two out of three dimensions of color can be shown in the chromaticity diagram. A three dimensional CIE color space is made by plotting axis Y rising from the illuminant point of the chromaticity diagram (Figures 2a, 2b). Therefore, the three-

dimensional chromaticity diagram can be formed by the usage of two chromaticity coordinates x , y and Y , where all the real colors lie within, with exception of the black color. While the color white is located on the top of an achromatic axis in this diagram, the color black can lie anywhere because it is not well defined through the mathematical definition where X , Y , Z tristimulus values are all zero and corresponding x and y coordinates can be anywhere within the diagram.⁷

B. Opponent-Type Systems

There have been several proposals made in transforming the CIE tristimulus system. In 1939, Breckenridge et al.¹⁸ developed uniform chromaticity diagrams in which neutral colors, including the reference white, would plot at the intersection of the diagram's coordinate axes. The position of a color relative to the origin would indicate its hue and chroma (saturation). Later, Hunter^{6,7,19} adopted this concept and improved it step by step until he proposed the L , a , b color space in 1948¹⁹. L represents lightness, a represents redness or greenness and b represents blueness or yellowness. The Hunter L , a , b system achieved significant popularity by some industries (Figure 3).^{6,7,19,20}

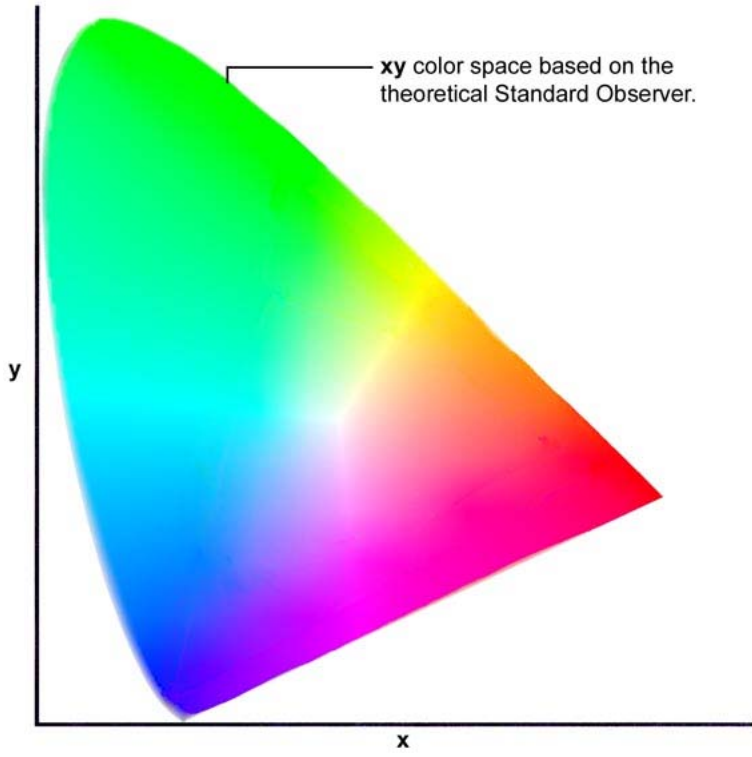


Figure 1. Chromaticity diagram.

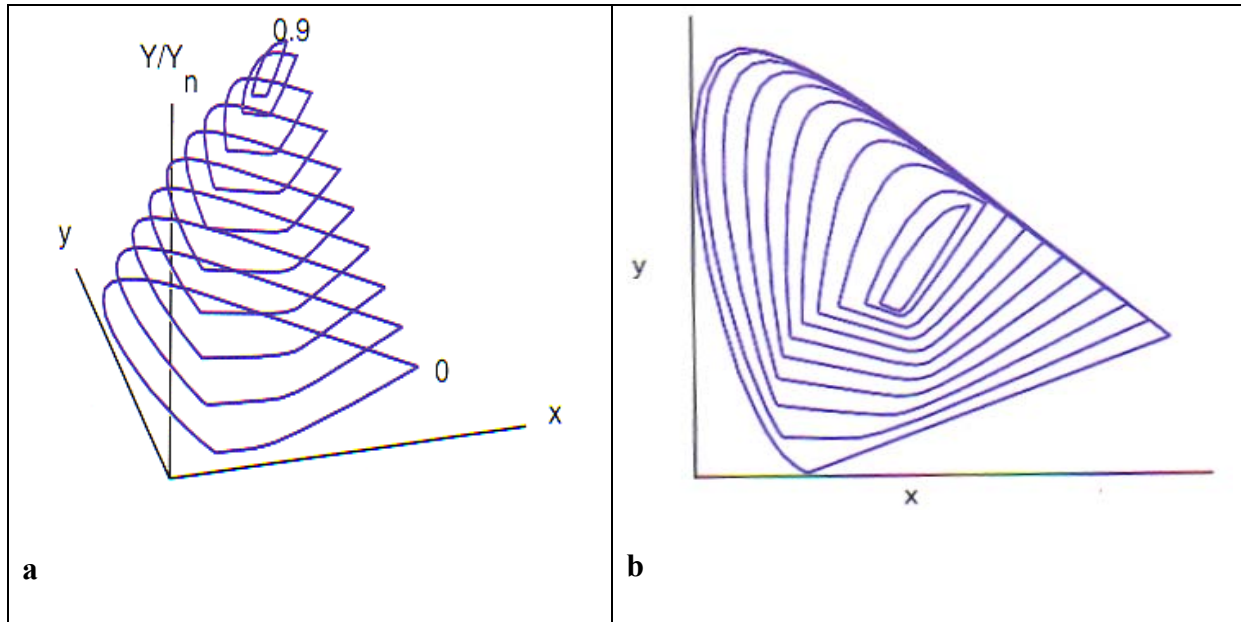


Figure 2. a. Added Y on chromaticity diagram,

b. Added Y on chromaticity diagram (looking down).

*Adapted from R.S. Berns, *Principles of color technology*, (John Wiley and Sons, New York 2000).

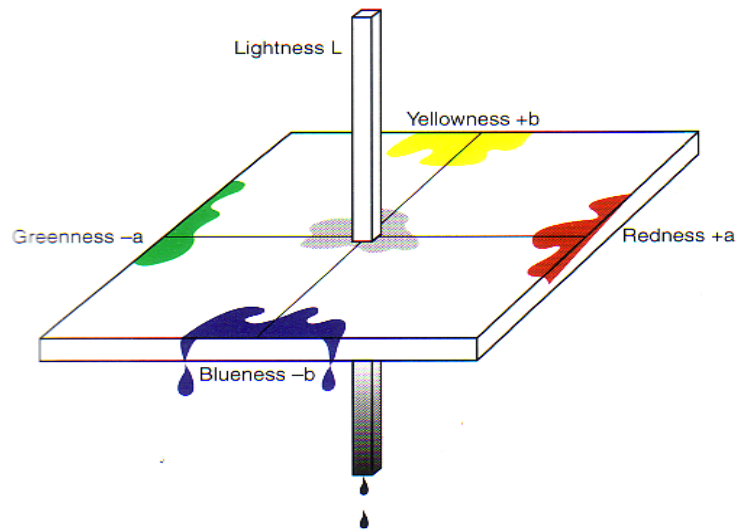


Figure 3. L, a, b color space.

*Adapted from R.S. Berns, *Principles of Color Tech.*, (John Wiley and Sons, New York 2000).

Although L, a, b coordinates are opponent coordinates in the Hunter system, they are not derived from opponent theory. Adams²¹ chromatic-value space was based on a Hering²² type opponent color vision theory. Nickerson²³ et al. modified Adams' chromatic value space by optimizing constants for their own color-difference data, rearranging one of the chromatic axes to result in opponent properties similar to Hunter L, a, b. With the suggestions by Glasser,²⁴ the ANLAB equation was formulated. In 1976, the CIE²⁵ introduced two new transformations of the chromaticity diagram that were modified to the ANLAB space for easier calculation. They are the L*, a*, b* system (CIELAB) and the L*, u*, v* system (CIELUV). L* again indicates lightness, a* and u* indicate redness or greenness, and b* and v* indicating blueness or yellowness. Cylindrical polar coordinates; CIELC_{ab}H_{ab} and CIELC_{uv}H_{uv}, were also defined by CIE. Both CIELC_{ab}H_{ab} and CIELC_{uv}H_{uv} correlate with lightness, chroma and hue, respectively, while only C_{uv} correlates with saturation. This is because as CIE defined saturation, which is derived from the chromaticity diagram, it is defined only for CIELUV.

Nevertheless, the ratio C_{ab}*/L* is sometimes assumed to correlate with saturation. Both systems were modified for the CIE standard observer 1964 (10°) and illuminant where for the illuminant L* is always 100, a* (u*) and b*(v*) are always 0. In general, usage of the two equations is not recommended when the illuminant is much different from average daylight. Today, both systems, as well as Hunter L, a, b are frequently used in color matching of materials. However, it should be noted that there are still no perfect color space systems. None of the above-explained systems provide equal visual differences as equal distances on the color diagram for all regions of the color space.^{1,7}

C. One Dimensional Color Spaces

Color properties like hue, lightness, and saturation have been used to describe a single color

property. Consequently, there have been many systems developed to describe the color of a variety of objects with a single number over the years. Frequently these objects differ in a single respect, such as saturation, and their color varies with this amount over a comparatively narrow and well-defined range.^{1,7}

Describing a particular color property usually entails visual or instrumental comparison of the test object to a series of standards. If the selected standards have color and spectral reflectance or transmittance curves similar to those of the test object, the comparison is easily possible and the position of the test object on the scale can be easily and precisely established. In cases like that, one-dimensional scales (single-number) are useful. One very important condition for a precise, accurate one-dimensional scale is that the test sample must be similar in both color and spectral properties to the set of standards used to calibrate and maintain the scale. As long as this condition is satisfied, one-dimensional or single-color scales work reasonably well. If the test sample differs in color and/or in spectral properties from that of the standards, then it becomes very difficult, if not impossible, to make a dependable judgment of the position of the sample on the scale. In this case, the test sample and the standard form a metameric pair and one-dimensional color scale may produce erroneous and misleading results.^{1,7}

D. Whiteness Scales

White is the achromatic object color of greatest lightness, characteristically perceived to belong to objects that reflect diffusely nearly all the incident energy throughout the visible spectrum.^{18,26}

Whiteness is associated with a region in color space where objects are recognized as being white.^{1,27} The degree of whiteness is measured by the degree of departure of the object from a perfect white. As noted above, there have been disagreements on what perfect white is and which directions of departure from it should be preferred or avoided.¹⁻³ In addition, usage of fluorescent

whitening agents to improve whiteness of objects adds to the discrepancies.¹⁻⁵ Whiteness of a material with a fluorescent whitening agent strongly depends on the spectral properties of the illumination for both visual evaluations and instrumental measurements. For these reasons, no single formula has been widely accepted for whiteness.^{1-3,12}

E. Whiteness

Whiteness is an attribute of colors of high luminous reflectance and low purity, situated in a relatively small region of the color space. The color white is distinguished by its high lightness and its very low (ideally zero) saturation. As stated earlier, when judging whiteness it is felt to be more attractive with a blue cast, rather than yellow cast, at a comparable luminance.^{3,28,29} Depending on the hues of near whites, the perception may differ. For example, an object with a blue cast will be perceived whiter than an object that has a yellow cast, where saturation and lightness are the same for both objects.^{27,28} Whiteness depends on observers and observer preferences.⁸ Visual assessment is impaired by individual preferences. Instrumental whiteness assessment will be absolute only if the uncertainties of measurement and of evaluation are overcome.⁹ The concept also depends on the assessment methods applied such as, ranking, pair comparison, difference scaling, and ratio scaling for a particular observer. Many varying conditions such as the level and spectral power distribution of sample irradiation, the color of the surrounding whites, and the desired appearance of various products have also direct impact on the whiteness perceived⁸. The intensity of fluorescence of FWA (Fluorescent Whitening Agent) added samples, for example, depends on the spectral power distribution of the illumination, especially in the UV region. Differences among visual assessments, measurements and different measurement devices also result from differences in spectral power distribution of the illumination.⁹ On the other hand, the general agreement is that samples are considered less white

or darker if they are yellower and darker.⁸

According to Ernst Ganz,⁹ three uncharacteristic observations are made if samples are compared where they differ only moderately in whiteness:

1. Different observers may give different weights to lightness and blueness. A sample can be ranked for its whiteness, although a difference in lightness and/or blueness may be clearly perceived.
2. Some observers prefer whites with a greenish while some prefer reddish tint. This causes contradictory evaluations of whiteness values where there is a hue difference in a given sample. At the same time samples with an intermediate bluish or neutral tint are assessed more consistently.
3. There is in general agreement among observers for hue differences, which is assessed independent of the perceived difference in whiteness.

These three effects could explain why samples with different luminous reflectance, purity, and dominant wavelength can be ranked linearly for their perceived whiteness values under given conditions by observers, although no general agreement on whiteness can be reached.⁸

It is important to note that whiteness must be defined based on perceptual evaluations and psychometric techniques before instrumentally measuring whiteness. In reality, the appearance of an object is evaluated by the observer. Therefore, an objective measure cannot be built until the subjective reality has been analyzed.³⁰ Instrumental measurement of whiteness simulating a psychometric scale is accompanied by an additional term that indicates to what extent and in what direction the appearance of the sample deviates from the maximum whiteness.³⁰

F. Whiteness Formulas

Over the years, many studies have been carried out and many formulas proposed. It has been realized through the studies that whiteness is entirely based on experience with the reflection of light of all wavelengths and that no measurement with a single filter in any particular region of the spectrum can ever be sufficient as a whiteness measurement. The visual experience of whiteness is done through color assessments and the instrumental measurement of whiteness is based on color measurement.¹

In whiteness measurements, the problem is to develop a single formula that gives an appropriate weighting to the tristimulus values. Many formulas have been developed, most of them have attempted to add the X, Y, Z tristimulus values in different algebraic proportions in order to achieve a weighting towards the blue region corresponding to subjective experience.³ Another problem is that the perfectly reflecting diffuser is not perfectly white. The human eyes, in combination with the brain, classify white that is slightly bluer than an object that reflects perfectly over the whole visual range.^{1,29,31} Selection of a single whiteness formula would not be sufficient, since the perception of the appearance of an object varies from one individual to another. In addition, there is no general agreement in the evaluation of whiteness.⁹

Whiteness evaluations made at uniform chromaticity spacing are less problematic, because only a small part of the color space is taken by white colors. There, chromaticity spacing correlates well with human perception.⁹

Whiteness determined by colorimetric measurements through a whiteness formula is not an exact quantity. Its value depends on the spectrophotometer used, properties of the whiteness formula used and illumination state. Therefore, differences in whiteness values are considered significant, only if the same instrument is used for the measurements.³⁰

Since it is not possible for a producer to fabricate a stable source that has the relative power distribution of D_{65} with adequate accuracy, there have been methods developed to convert to illuminant D_{65} through the use of the spectral radiance factors that are measured with a source of different, but known, relative spectral power distribution.

A device for controlling the relative UV contents of the sample irradiation for use by spectrophotometers attained an approximation to a stable relative spectral power distribution. The excitation of the fluorescence of a stable white reference sample is attenuated sufficiently to keep the tristimulus values of this sample constant with an adjustable UV filter.⁹

Attempts have been made to standardize the calculation of whiteness, although colorimetry of fluorescent samples is still problematic.¹⁻³ Most formulas used today assess a sample relatively as to whiter, lighter and bluer. When these formulas are applied to colored samples they are generally meaningless. On the other hand, they satisfactorily characterize the appearance of commercial whiteness. They are mostly unsuccessful in assessing tinted samples with chromaticities placed on the borders of white colors.^{32,33} However, there are formulas developed for dealing with such cases. They produce ellipses that are equivalent lines that should be centered on the unknown preferred white in the chromaticity chart. This point is not reached under normal viewing conditions, because of lower UV content of the illumination source in comparison to daylight. This limits the excitation of fluorescence.²¹

Any whiteness formula defines surfaces of constant whiteness in the color space and they are based on luminous reflectance Y and on transformed chromaticity coordinates, such as colorimetric saturation.⁹

While whiteness is a qualifying assessment, tint deviation is a descriptive assessment and thus not a simple quality judgment. Tint deviation describes the hue of the sample in comparison

with the equally white step of the white scale to which the formula parameters relate. Therefore, tint deviation is mainly a descriptive assessment and does not entail quality judgment. It directly depends on the tint of the white scale being used as the reference.¹³ The tint is assessed visually and chosen by colorimetric means, because the whiteness value does not characterize a white by itself. The influence of the relative UV content in the measurement on the evaluation of tint is negligible.

G. CIE Whiteness

Beginning in 1981, CIE started extensive studies to solve the problem created by the excess of equations available. Using results from Ganz,^{8,9} in 1981 the CIE recommended an equation for the whiteness W , related to basic CIE tristimulus measurements, and having the form:^{1,10,11}

$$W_{\text{CIE}} = Y + 800(x_n - x) + 1700(y_n - y), \quad (2)$$

where x and y are the CIE chromaticity co-ordinates and x_n and y_n are the co-ordinates for the perfect reflecting diffuser at the given illumination. For example, at $D_{65} 10^\circ$, $x_n = .313795$ and $y_n = .330972$.¹⁴

This equation is complemented by the tint equations:

$$T = 1000(x_n - x) - 650(y_n - y), \quad \text{for a } 2^\circ \text{ observer} \quad (3a)$$

or

$$T = 900(x_n - x) - 650(y_n - y), \quad \text{for a } 10^\circ \text{ observer} \quad (3b)$$

These give tint-values in the red or green direction for the 1931 or 1964 CIE standard observer, respectively. A positive value of T indicates greenness and a negative value indicates redness.

These equations can be used only in a limited region. Criteria for whiteness are that the values of W fall within the limits given by:

$$5Y-280 > W_{CIE} > 40 \quad (4a)$$

and the tint value T shall fall within the limits given by:

$$3 > T > -3 \quad (4b)$$

The W formula describes an axis in the blue-yellow direction with a dominant wavelength of 470 nm^{3,13,14} in the CIE chromaticity diagram and the inequalities limit the extent to which a sample may enter the blue or yellow regions or stay towards the red or green and still be classified as white.⁸⁻¹¹ According to this definition, the perfect reflecting diffuser has a whiteness of 100 and a zero tint value.

One clear disadvantage in CIE whiteness is that this system of equations does not clarify whether the whiteness has any component of blueness or yellowness. A standard measure for yellowness is given by ASTM E-0313 or DI-1925.^{1,3,34,35}

$$YI = 100(C_x X - C_z Z)/Y, \quad (5)$$

where $C_x=1.301$ ($D_{65} 10^\circ$) or 1.277 ($C 2^\circ$) and $C_z=1.150$ ($D_{65} 10^\circ$) or 1.059 ($C 2^\circ$). In this equation, zero as given by this equation, is associated with the visual zero between bluishness and yellowness.

The CIE suggests that the formula should only be used for relative evaluations and these are valid only for measurements with a single instrument at a given time and without reference to a white scale. Evaluations with the formulas are significantly improved, if the sample illumination is stabilized and fitted as closely as possible to a desired illuminant.^{13,14} This also improves the matching of different measuring instruments for whiteness. The tint deviation or hue value can still not be adequately matched.¹⁴

Whiteness and tint formulas proposed by the CIE are restricted to samples differing not too broadly in tint and fluorescence. The measurements have to be executed with the same

instrument at about the same time. The formulas produce relative, not absolute, white assessments seemingly adequate for commercial uses in many cases. Again, the measuring instruments must have illumination resembling daylight.^{9,27}

If the sample illumination is stabilized, assessments with the CIE formulas are significantly improved and samples to be compared do not have to be measured at the same time. This also improves the matching of different measuring instruments for whiteness. The tint deviation or hue value can still not be adequately matched.^{13,14}

H. Contributions of Luminance Factor and of Chromaticity to Whiteness

Color information that is independent of the luminance or luminance factor is called chromaticness³⁶, hence the chromaticity diagram. However, chromaticities should correlate to some extent with the stimulus hue, and chroma (saturation).^{1,2}

Other than whiteness formulas and tint values, the third value characterizing a white sample is the luminance factor Y .⁹ This kind of whiteness can be characterized by the contribution of chromaticity.

$$C=W-Y \quad (6)$$

where $C > 0$ and $W > Y$ for bluish whites and

$C < 0$ and $W < Y$ for yellowish whites

To prevent application of whiteness formulas to colored samples the following limits were proposed:⁹

$$Y > 70 \quad (7a)$$

$$W > 40 \quad (7b)$$

$$-6 < T < 6 \quad (7c)$$

Equation 2 was proposed as a standard whiteness formula of neutral hue preference.

Two supplementary whiteness formulas, one for green hue preference

$$W = Y + 1700(x_n - x) + 900(y_n - y) \quad (8a)$$

and one for the red hue preference

$$W = Y - 800(x_n - x) + 3000(y_n - y) \quad (8b)$$

are required to cover all evaluations.⁹

The above three formulations can be used with colorimetric data evaluated for standard illuminant D₆₅ and both CIE 1931 2° and CIE 1964 10° observers.

Two standard tint formulas were proposed for a D₆₅ illuminant, one each for the CIE 1931 2° and CIE 1964 10° standard observers⁷ respectively, were proposed.

$$T = -1000(x - x_n) + 1700(y - y_n) \quad (9a)$$

$$T = -900(x - x_n) + 800(y - y_n). \quad (9b)$$

I. Whiteness Region in the Chromaticity Diagram for CIE Whiteness

As mentioned earlier, whiteness can be thought of as a region in the chromaticity diagram. This region is a quadrilateral having four corner points. The x and y values at the corner points in the chromaticity diagram can be calculated for a given Y value by using CIE tint and whiteness inequality conditions (inequalities $-3 < T < 3$).

Applying the tint condition **9a and 9b**, we have

$$(20/13)(x - x_n) + 3/650 > y - y_n > (20/13)(x - x_n) - 3/650 \text{ for } 2^\circ \quad (10a)$$

$$(18/13)(x - x_n) + 3/650 > y - y_n > (18/13)(x - x_n) - 3/650 \text{ for } 10^\circ \quad (10b)$$

For $x = x_n$, these both become

$$3/650 > y - y_n > -3/650 \quad (11)$$

From the Whiteness condition **4a**, we have

$$(280 - 4Y)/1700 < y - y_n + (8/17)(x - x_n) < (Y - 40)/1700 \quad (12)$$

For $x=x_n$ and $Y=100$, this becomes

$$-6/85 < y-y_n < 3/85 \quad (13)$$

If the intersection points for x and y are calculated for a given Y value, the whiteness region can accurately be determined. For the 10° observer, this is

$$y-y_n = (18/13)(x-x_n) \pm 3/650 \quad (14a)$$

$$y-y_n + (8/17)(x-x_n) = (Y-40)/1700 \quad (14b)$$

$$(280-4Y)/1700 \quad (14c)$$

After solving these 4 equations, we obtain 4 values of $x-x_n$,

$$x-x_n = 13(Y-40)/41000 - 51/20500 \quad (15a)$$

$$= 13(Y-40)/41000 + 51/20500 \quad (15b)$$

$$= 13(280-4Y)/41000 - 51/20500 \quad (15c)$$

$$= 13(280-4Y)/41000 + 51/20500 \quad (15d)$$

and

$$y-y_n = (18/13)[13(Y-40)/41000 - 51/20500] + 3/650 \quad (16a)$$

$$= (18/13)[13(Y-40)/41000 + 51/20500] - 3/650 \quad (16b)$$

$$= (18/13)[13(280-4Y)/41000 - 51/20500] + 3/650 \quad (16c)$$

$$= (18/13)[13(280-4Y)/41000 + 51/20500] - 3/650 \quad (16d)$$

Similar results can be obtained for the 2° observer.

For $Y=100$, the four corner points within the whiteness region are found to be:

$$(x-x_n, y-y_n) = (0.02151, 0.02517) \quad (17a)$$

$$(-0.01654, 0.02751) \quad (17b)$$

$$(-0.03556, -0.05385) \quad (17c)$$

$$(-0.04054, -0.05151) \quad (17d)$$

For these,

$$\begin{aligned} W_{\text{CIE}} &= Y+800(x_n - x)+1700(y_n - y) \\ &= 40,40,220,220 \end{aligned} \quad (18)$$

For D₆₅, 10° the four corners are:

$$(W_{\text{CIE}}, C^*) = 40, 13.8 \quad 40, 14.5 \quad 220, 30.3 \quad 220, 29.0 \quad (19a)$$

$$(x, y) = (0.3353,0.3561) \quad (0.3330,0.3585) \quad (0.2782,0.2771) \quad (0.2733,0.2795) \quad (19b)$$

$$(L^*, a^*, b^*) = (100,-1.2,13.8) \quad (100,-4.7,13.7) \quad (100,10.2,-28.7) \quad (100,5.4,-28.5) \quad (19c)$$

where $C^*=(a^{*2}+b^{*2})^{1/2}$ is the Chroma.

Likewise for D₅₀, 10°:

$$(W_{\text{CIE}}, C^*) = 40, 15.5 \quad 40, 15.8 \quad 220, 32.1 \quad 220, 31.1 \quad (20a)$$

$$(x, y) = (0.3692,0.3846) \quad (0.3642,0.3870) \quad (0.3121,0.3056) \quad (0.3072,0.3080) \quad (20b)$$

$$(L^*, a^*, b^*) = (100,-1.3,15.5) \quad (100,-4.5,15.2) \quad (100,9.2,-30.7) \quad (100,5.1,-30.7) \quad (20c)$$

These results point out the weakness in the CIE Whiteness formula. Clearly, all of these corner points of the “whiteness” region are far from white by any sensible measure. Relatively saturated chroma values (>30) are associated with the maximum whiteness values. Such results are clearly absurd. Similar conclusions are reached, if we use more moderate Y values.

For example, many commercial papers, such as publication grades, have Y approximately 83 or greater. Thus, the corresponding calculated values for Y=83 are:

for D₆₅, 10°

$$(W_{\text{CIE}}, C^*) = 40, 9.3 \quad 40, 9.9 \quad 135, 12.3 \quad 135, 11.4 \quad (21a)$$

$$(x, y) = (0.3299,0.3487) \quad (0.3249,0.3510) \quad (0.2998,0.3070) \quad (0.2948,0.3093) \quad (21b)$$

$$(L^*, a^*, b^*) = (93.0,-.3,9.3) \quad (93.0,-3.7,9.2) \quad (93.0,4.7,-11.4) \quad (93.0,.8,-11.4) \quad (21c)$$

and at D₅₀, 10°

$$(W_{CIE}, C^*) = 40, 10.4 \quad 40, 10.8 \quad 135, 13.1 \quad 135, 12.5 \quad (22a)$$

$$(x, y) = (0.3638, 0.3772) \quad (0.3589, 0.3795) \quad (0.3337, 0.3355) \quad (0.3287, 0.3378) \quad (22b)$$

$$(L^*, a^*, b^*) = (93.0, -4, 10.4) \quad (93.0, -3.5, 10.2) \quad (93.0, 4.4, -12.4) \quad (93.0, 9, -12.4) \quad (22c)$$

Chroma values more than 13 for the high CIE whiteness corners are obviously far from white. Clearly, a new Whiteness formula is needed to avoid such results.

The four positions at each white point for Y=100, that define the white boundary are shown in Figures 4a and 4b. The planes in the whiteness region for a 5 point incremental change in Y from 100 are also shown in Figures 5a and 5b. The range of Y in commercial printing papers can be between 70 and 100.

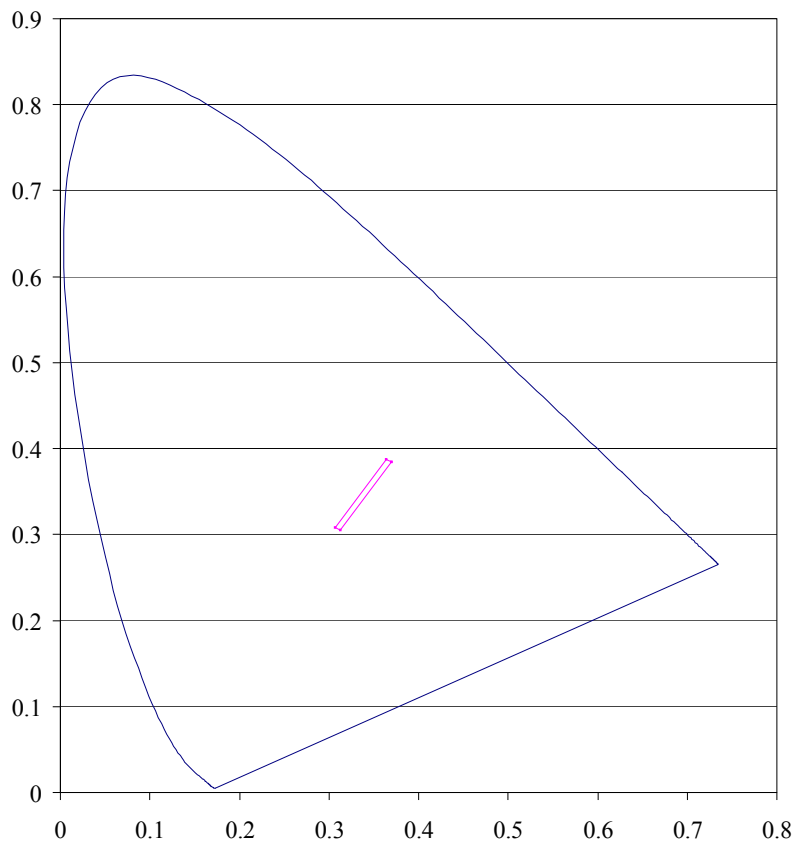


Figure 4a. Four corner positions at each white point for Y=100 at D50/10°.

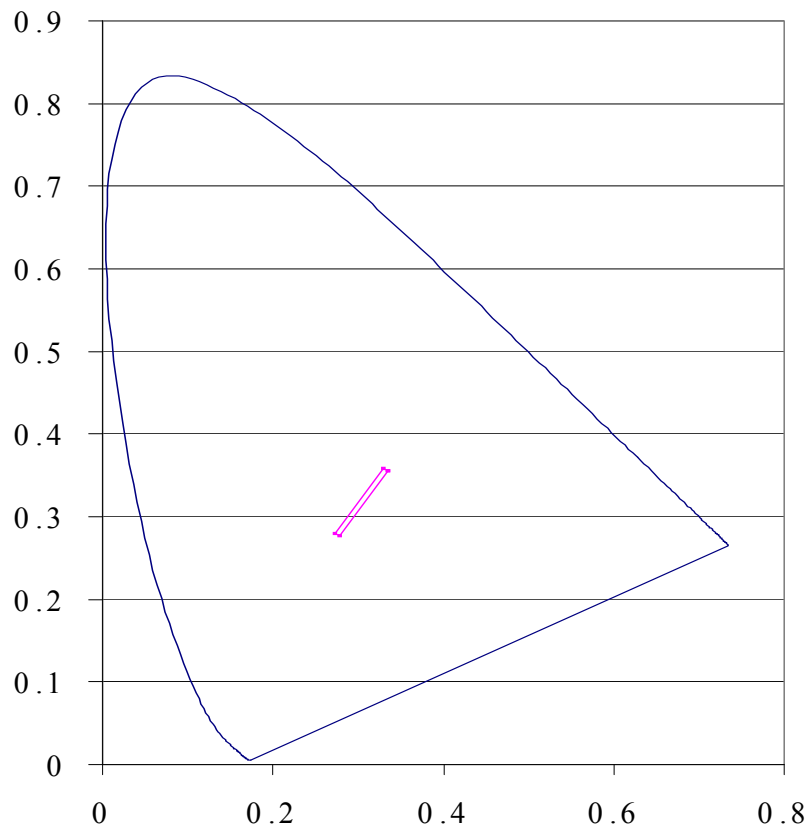


Figure 4b. Four corner positions at each white point for $Y=100$

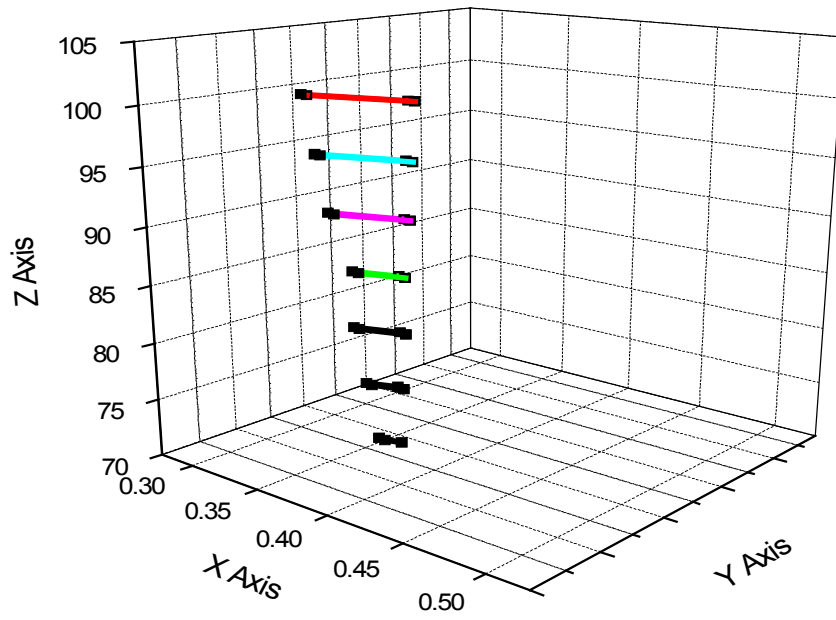


Figure 5a. Planes in the white region at varying Y values at D50/10°.

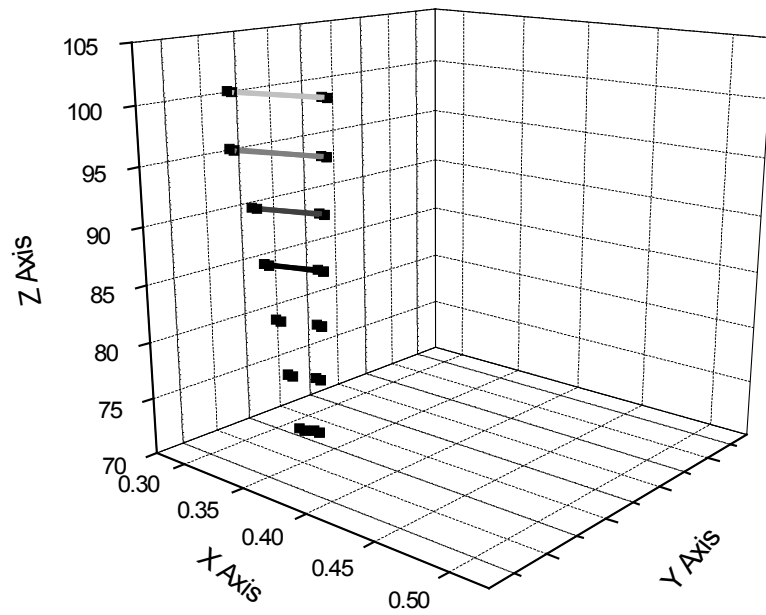


Figure 5b. Planes in the white region at varying Y values at D65/10°.

3. NEW WHITENESS FORMULAS

Previously, we showed^{32,33} that large differences in calculated CIE Whiteness can frequently occur, when the colors of papers are slightly altered from neutral white to varying shades of whites. In the same study, observers assessed the whiteness of the tinted papers as either equivalent or very similar in appearance to one another.

Here we propose new formulas that will reduce the large calculated differences among slightly tinted papers, within the whiteness region in the chromaticity diagram. They should establish a better correlation between the calculated values and perceptual visualization assessments. One formula will measure nearness to neutral white, while the second will characterize whiteness in the sense discussed above. That is, high whiteness is associated with high luminescence and moderate saturation with a blue cast.

For nearness to neutral white, the Luminance factor L and Chroma (related to saturation) are selected as the appropriate variables in this formula, since whiteness is directly dependent on and characterized by these two properties. An exponential decay function is proposed to reduce large differences in calculated whiteness values in slightly colored papers.

For nearness to neutral white, we propose a formula of the form,

$$N_{FA} = L * e^{-\lambda(C/C_0)^2} \quad (23)$$

where C_0 is a characteristic chroma value, presumably related to the saturations at the four corners of the whiteness region and λ is a coefficient to be determined.

For $C = C_0$, $N_{FA} = L * e^{-\lambda}$ if we assume $e^{-\lambda} = 1/2$, then $\lambda = \ln 2 = 0.693$, this yields the working form for our Neutrality formula:

$$N_{FA} = L * (1/2)^{(C/C_0)^2} \quad (24)$$

For whiteness, we will employ a similar form. We propose a whiteness formula of the form

$$W_{FA} = W_0 e^{-\lambda[(a^*-a_1^*)^2 + (b^*-b_1^*)^2]/C_2} \quad (25)$$

where W_0 , a_1^* and b_1^* are to be determined and C_2 is a characteristic chroma, possibly different from C_0 .

We will determine W_0 , a_1^* and a_2^* by requiring that W_{FA} be equal to W_{CIE} from equation 2 for $a^*=b^*=0$ and the derivatives with respect to a^* and b^* at the origin be equal to those obtained from equation 2. In order to do this, we must express W_{CIE} in terms of a^* and b^* . Using the definitions of CIELAB²³ in terms of the tristimulus values¹³, we obtain

$$W_{CIE} = Y + (a^*W_a - b^*W_b)/D \quad (26)$$

where

$$W_a = \alpha[3(Y/Y_n)^{2/3} + 3(Y/Y_n)^{1/3}a^*/500 + (a^*/500)^2]/3 \quad (27a)$$

$$W_b = \beta[3(Y/Y_n)^{2/3} - 3(Y/Y_n)^{1/3}b^*/200 + (b^*/200)^2]/3 \quad \text{and (27b)}$$

$$D = Y/Y_n + x_n a^*/500 [3(Y/Y_n)^{2/3} + 3(Y/Y_n)^{1/3} a^*/500 + (a^*/500)^2] - z_n b^*/200 [3(Y/Y_n)^{2/3} - 3(Y/Y_n)^{1/3} b^*/200 + (b^*/200)^2] \quad (27c)$$

where $\alpha = 3x_n(900y_n - 800z_n)/500$ and $\beta = 3z_n(800x_n + 1700y_n)/200$.

Equations 26 and 27 should be compared with the expression obtained by Ganz and Pauli³⁷:

$$W_{CIE} = 2.41L^* - 4.45b^*[1 - 0.0090(L^* - 96)]^4 - 141.4 \quad (28)$$

which was determined by regression analysis of the CIE whiteness formula for D_{65} , 10° with respect to a^* and b^* . Equation 26 is rigorous and is equivalent to the usual CIE formula.

We see from equation 26 that for $a^*=b^*=0$:

$$W_{CIE} = Y \quad (29a)$$

$$\partial W_{CIE}/\partial a^* = W_a/D = \alpha(Y/Y_n)^{-1/3} \quad (29b)$$

$$\partial W_{CIE}/\partial b^* = -W_b/D = -\beta(Y/Y_n)^{-1/3} \quad (29c)$$

For D_{65} , 10° , the derivatives become

$$\partial W_{CIE}/\partial a^* = 0.0258(Y/Y_n)^{-1/3} \quad (30a)$$

$$\partial W_{CIE}/\partial b^* = 4.336(Y/Y_n)^{-1/3} \quad (30b)$$

If we expand Y to first order in L-96, we obtain for W near $a^*=b^*=0$

$$W_{CIE} = 2.411L^* + (0.02670a^* - 4.490b^*)[1-0.00893(L^* - 96)] - 141.448 \quad (31)$$

in good agreement with equation 28. As implied by Ganz and Pauli's regression, the a^* term is negligible for practical purposes.

The corresponding derivatives of equation 25 for $a^*=b^*=0$ are

$$\partial W_{FA}/\partial a^* = 2\lambda a_1^* Y/C_2^2 \quad \text{and (32a)}$$

$$\partial W_{FA}/\partial b^* = 2\lambda b_1^* Y/C_2^2 \quad (32b)$$

Equating equations 29b and 29c to 32a and 32b, respectively, we obtain

$$W_0 = Y e^{\lambda(C_1/C_2)^2} \quad (33a)$$

$$a_1^* = \alpha(Y_n/Y)^{4/3} C_2^2 / (200\lambda) \quad \text{and (33b)}$$

$$b_1^* = -\beta(Y_n/Y)^{4/3} C_2^2 / (200\lambda) \quad (33c)$$

where $C_1 = (a_1^{*2} + b_1^{*2})^{1/2} = \gamma C_2^2 / (200\lambda) (Y_n/Y)^{4/3}$, with $\gamma = (\alpha^2 + \beta^2)^{1/2}$. Setting $\lambda = \ln 2$, as before, we end up with

$$W_{FA} = Y (1/2)^{[a^*(a^*-2a_1^*)+b^*(b^*-2b_1^*)]/C_2^2} \quad (34)$$

The W_{FA} and CIE whiteness equations are shown in Figures 6a and 6b and compared in Figure 6c in L, a^* , b^* color space for L=93.

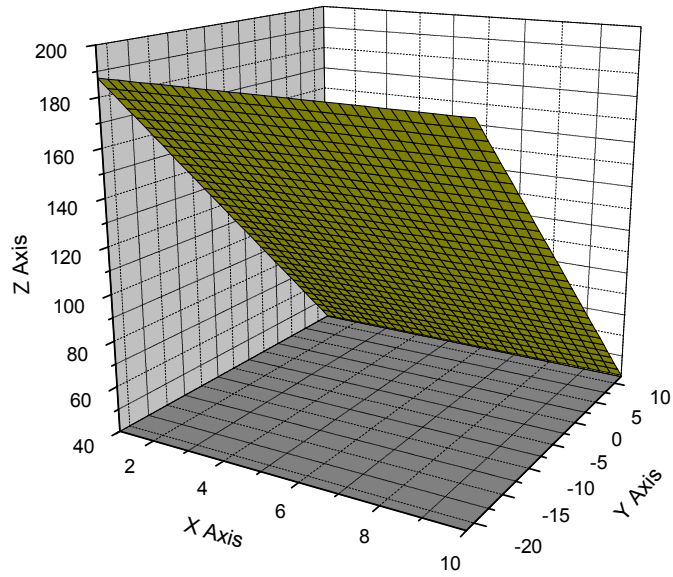


Figure 6a. CIE equation on L, a*, b* color space.

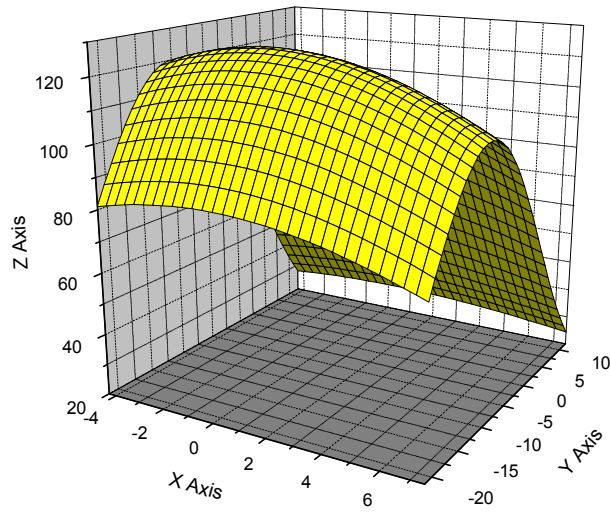


Figure 6b. W_{FA} equation on L, a*, b* color space.

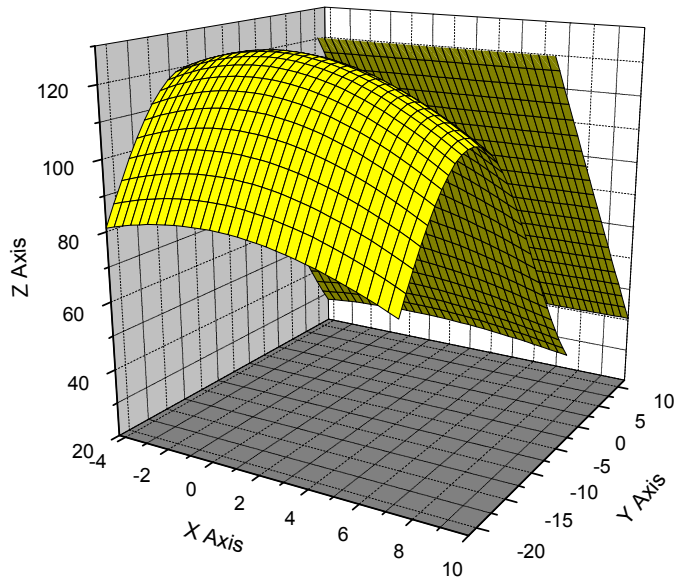


Figure 6c. Comparison of CIE and W_{FA} equation on L^* , a^* , b^* color space.

The characteristics of this formula are that it has a maximum value at a_1^* and b_1^* and decays to zero far away from the maximum. With the exponential form, we don't need to explicitly include the tint formulas, because the exponential forms already reduce the values for tinted samples. Unlike the CIE and other whiteness formulas, it is a smooth function and doesn't cut off abruptly at the inequality limits. This is consistent with the known continuity and smoothness of the visible color space. Reducing the whiteness below a maximum value is consistent with the belief that increasing the blueness of a sample (reducing the b value) improves whiteness up to a point, after which the apparent whiteness decreases.

The remaining task is to determine the characteristic chroma values C_0 and C_2 . They represent the distance in a^* - b^* space at which the functions decay to half their maximum value. Their values should, in some sense, represent the area of the whiteness region for given Y (or L^*) values. They should be related to the corner saturations calculated from equations 15 and 16. In what follows, we will set both C_0 and C_2 to the maximum corner chroma value. This

corresponds to a large region of applicability, while discounting samples far from the maximum value. Application of these formulas will be illustrated in the following section.

EXPERIMENTAL PROCEDURE

Three commercial digital printing papers were chosen for testing of our formulas. These were Xerox Phaser 860/8200 Glossy coated paper (145 gsm), Stora Enso 4CC Silk (130 gsm) and Stora Enso 4CC Cover (160 gsm). The Phaser paper has $Y = 84.9$ ($L^* = 93.8$), $a^* = 0.7$ and $b^* = -2.5$ at $D65\ 10^\circ$, as measured on a GretagMacbeth SpectroScanT spectrophotometer in reflection mode. Likewise, the Silk paper has $Y = 89.0$ ($L^* = 95.6$), $a^* = .9$ and $b^* = -3.9$ and the Cover has $Y = 86.4$ ($L^* = 94.5$), $a^* = 2.2$ and $b^* = -9.4$. ICC^{38,39} output profiles⁴⁰ were determined for these three papers on a Xerox Phaser 8200 solid ink printer, by measuring an ECI 2002R⁴¹ output test chart with the SpectroScanT in reflection mode, and calculating the profile with GretagMacbeth ProfileMaker. ProfileMaker is known to produce accurate ICC profiles.⁴²⁻⁴⁴

Using the Xerox paper, as close as possible to the four corner points, a neutral point and a near to maximum W_{FA} point were printed for a nominal Y of 83 ($L = 93$). The corner values were chosen as closely as possible to the chromaticity values in Equations 21 or 22, within the gamut of the printer. This pattern is shown in Figure 7. The actual nominal and measured values of Hunter Lab and CIELAB are given in Table 1, along with the usual ΔE value define by

$$\Delta E = (\Delta L^{*2} + \Delta a^{*2} + \Delta b^{*2})^{1/2} \quad (35)$$

where ΔL^* , Δa^* and Δb^* are the differences between the nominal and measure values.

In addition, a modified version of Figure 7 was printed, where the Upper Middle and Lower Middle patches were interchanged.

In addition to the corners, a series of steps, with varied nominal b^* values were printed on the Silk and Cover papers, respectively. The corresponding nominal and measured Hunter Lab

and CIELAB values for these steps are given in Tables 2 and 3. Y values of the coated color samples in the chromaticity diagram are shown as in Figure 8.

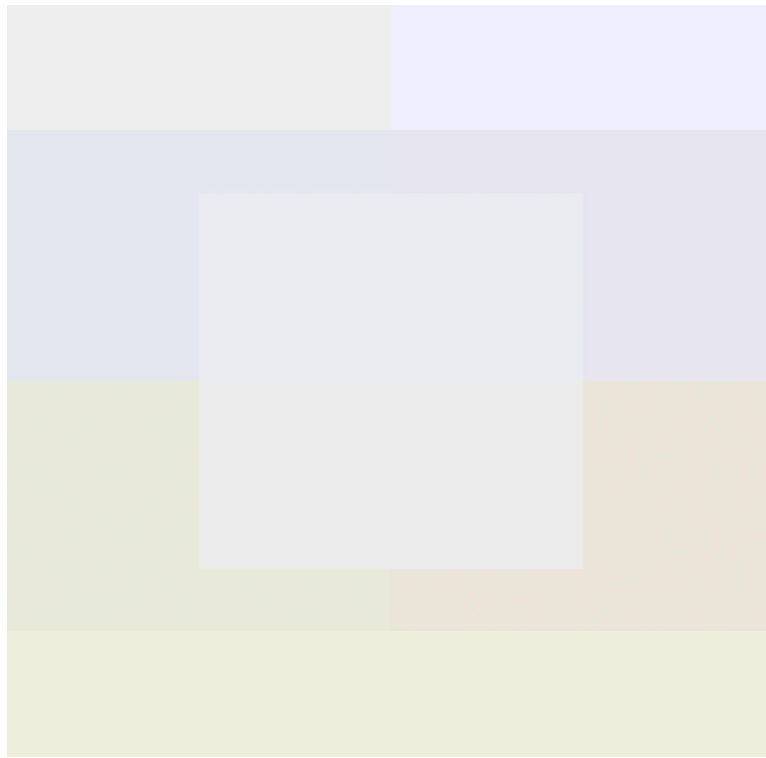


Figure 7. Pattern printed on Xerox Phaser 8200 printer for viewer observations.

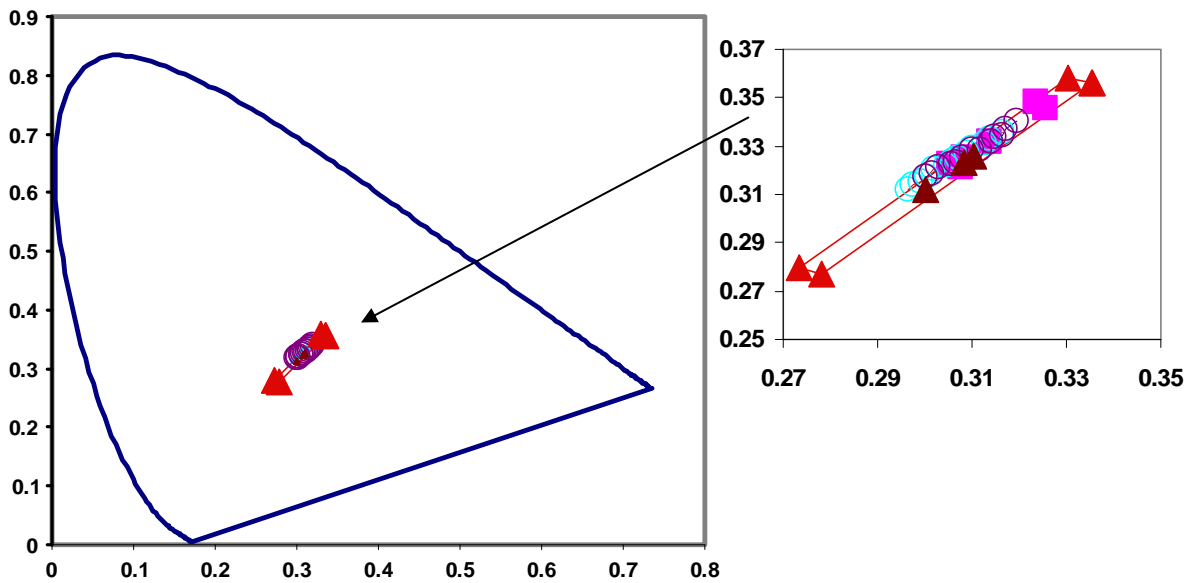


Figure 8. Coated color samples in the chromaticity diagram for their Y values.

In order to compare predictions of the test formulas, 48 randomly selected observers were asked to rank each printed patch for their visual appearance for whiteness. In addition, they were asked to judge the three unprinted papers for whiteness. Twenty-five of the observers evaluated the samples in a 5000 °K light booth and 23 of the observers under the daylight behind a window. We chose primarily untrained observers, because they are expected to be more representative of buyers of paper and print. Observer evaluations were made on the basis of the optical appearance of each specimen and specimen regions (how white they observed each region on the sample). These paper specimen and specimen regions were then ranked by their arithmetic average.

However, since they are untrained, they are likely to be more varied, and thus larger samplings are needed. Recall that the original CIE color matching functions¹³ were based on only 17 observers. Fewer trained observers might be required, but if they are to be trained according to a particular school of whiteness and formulas, it would interfere with the objectivity of the test.

Spectral reflectance values of each patch on the printed samples were measured with the GretagMacbeth SpectroScanT spectrophotometer. Tristimulus values (X, Y, Z), CIE, Hunter, Ganz and proposed whiteness formulas were calculated for D₅₀, 2° and C, 2°. In addition, CIE and HTAPPIunter L, a, b values for D₅₀, 2° (the standard reference for ICC profiles), and C, 2° and effective TAPPI⁴⁵ brightness were calculated from the measured spectra.

For each of the printed samples, L, L* and Y values were kept essentially constant for each printed patch, while a, b and a*, b* values were varied. The achieved L, L* values were in the range of 89-96 and the Y values were in the range of 79-89 for D₅₀, 2°, with similar values for D₆₅, 2° and C, 2°.

Table 1. Hunter L, a, b and L*, a*, b* Values Calculated from the Measured Spectra from Printed Figure 7 for D₅₀, 2°.

	Left				Right				Center			
	Upper		Lower		Upper		Lower		Upper		Lower	
	T ¹	M ²	T ¹	M ²	T ¹	M ²	T ¹	M ²	T ¹	M ²	T ¹	M ²
L	89.0	89.1	90.0	90.2	89.0	89.1	88.7	89.2	91.0	91.0	91.2	91.3
a	0.4	0.0	-3.0	-2.5	2.0	1.9	0.2	0.2	0.3	0.4	0.1	0.1
b	-4.1	-4.3	6.8	6.1	-4.0	-4.2	5.7	5.6	-2.6	-3.0	0.0	0.0
L*	91.3	91.4	92.1	92.3	91.3	91.4	91.1	91.5	92.9	93.0	93.1	93.2
a*	0.4	0.0	-3.0	-2.5	2.0	1.9	0.2	0.2	0.3	0.4	0.1	0.1
b*	-4.7	-4.9	8.4	7.5	-4.6	-4.9	7.0	6.9	-3.0	-3.5	0.0	0.0
ΔE	0.50		1.07		0.31		0.44		0.48		0.09	
RMS (ΔE)												0.57

¹T: Target

²M: Measured

Table 2. Hunter L, a, b and L*, a*, b* values calculated from the measured spectra from the printed steps on the Silk paper for D₅₀, 2°.

		L	a	b	L*	a*	b*	ΔE	RMS ΔE
Step 1	Target	89.5	0.3	-6.6	91.7	0.3	-7.5	0.79	
	Measured	88.9	-0.2	-6.9	91.2	-0.2	-7.8		
Step 2	Target	90.2	0.2	-6.1	92.3	0.2	-6.9	0.55	
	Measured	90.0	-0.2	-6.3	92.1	-0.2	-7.2		
Step 3	Target	91.7	-0.3	-5.1	93.5	-0.3	-5.8	0.52	
	Measured	91.5	-0.7	-5.3	93.4	-0.7	-6.0		
Step 4	Target	93.0	-0.1	-4.2	94.5	-0.1	-4.7	0.18	
	Measured	93.1	-0.2	-4.3	94.6	-0.2	-4.9		
Step 5	Target	93.4	0.4	-3.9	94.8	0.4	-4.4	0.18	
	Measured	93.5	0.3	-4.0	94.9	0.3	-4.5		
Step 6	Target	93.3	0.1	-2.9	94.8	0.1	-3.4	0.23	
	Measured	93.5	0.1	-3.1	94.9	0.1	-3.5		
Step 7	Target	93.2	-0.4	-1.5	94.7	-0.4	-1.7	0.35	
	Measured	93.3	-0.4	-1.8	94.8	-0.4	-2.0		
Step 8	Target	93.7	0.3	-1.2	95.1	0.3	-1.3	0.25	
	Measured	93.9	0.3	-1.3	95.2	0.3	-1.5		
Step 9	Target	93.7	-0.2	0.4	95.0	-0.2	0.5	0.32	
	Measured	93.8	-0.1	0.2	95.1	-0.1	0.2		
Step 10	Target	93.6	-0.4	1.0	95.0	-0.4	1.2	0.39	
	Measured	93.7	-0.3	0.7	95.1	-0.3	0.8		
Step 11	Target	92.9	0.0	1.6	94.4	0.0	1.9	0.37	
	Measured	93.1	0.1	1.3	94.6	0.1	1.5		
Step 12	Target	93.3	-1.0	2.6	94.8	-1.0	3.1	0.68	
	Measured	93.5	-0.7	2.1	94.9	-0.7	2.5		
Step 13	Target	93.1	-1.4	3.5	94.6	-1.4	4.2	0.33	
	Measured	93.4	-1.2	3.5	94.9	-1.2	4.2		

Table 3. Hunter L, a, b and L*, a*, b* Values Calculated from the Measured Spectra from the Printed Steps on the Cover Paper for D₅₀, 2°.

		L	a	b	L*	a*	b*	ΔE	RMS ΔE
Step 1	Target	91.1	1.1	-9.7	93.0	1.1	-10.8	0.51	
	Measured	90.6	0.8	-9.9	92.6	0.8	-11.0		
Step 2	Target	91.0	0.8	-8.8	92.9	0.8	-9.9	0.49	
	Measured	90.7	0.5	-9.1	92.7	0.5	-10.2		
Step 3	Target	91.2	0.9	-8.1	93.1	0.9	-9.0	0.45	
	Measured	91.0	0.7	-8.4	92.9	0.7	-9.4		
Step 4	Target	91.1	0.7	-7.3	93.0	0.7	-8.2	0.35	
	Measured	91.0	0.5	-7.5	92.9	0.4	-8.4		
Step 5	Target	90.7	-0.1	-6.1	92.7	-0.1	-6.9	0.30	
	Measured	90.6	-0.3	-6.2	92.6	-0.3	-7.0		
Step 6	Target	91.4	0.5	-5.0	93.2	0.5	-5.7	0.21	
	Measured	91.3	0.4	-5.1	93.2	0.4	-5.8		
Step 7	Target	91.3	0.2	-4.1	93.2	0.2	-4.7	0.18	
	Measured	91.3	0.1	-4.2	93.1	0.1	-4.8		
Step 8	Target	91.3	0.0	-3.5	93.1	0.0	-4.1	0.16	
	Measured	91.2	-0.1	-3.6	93.1	-0.1	-4.2		
Step 9	Target	91.4	0.0	-2.5	93.3	0.0	-2.9	0.11	
	Measured	91.5	0.0	-2.6	93.3	0.0	-3.0		
Step 10	Target	91.3	-0.1	-2.0	93.2	-0.1	-2.3	0.06	
	Measured	91.4	-0.2	-2.0	93.2	-0.2	-2.3		
Step 11	Target	91.6	0.1	-0.9	93.4	0.1	-1.1	0.23	
	Measured	91.8	0.1	-0.8	93.6	0.1	-0.9		
Step 12	Target	91.5	-0.2	-0.2	93.4	-0.2	-0.2	0.21	
	Measured	91.7	-0.1	0.0	93.5	-0.1	0.0		
Step 13	Target	90.9	0.0	0.8	92.8	0.0	1.0	0.48	
	Measured	91.1	0.0	1.2	93.0	0.0	1.4		

4. RESULTS AND DISCUSSION

Figure 9 shows the whiteness evaluation of the 48 observers for the printed pattern in Figure 7 and its variation. As seen from the Figure, an overwhelming number of 46 observers assessed one of the center areas as the whitest. One of these areas is nearly neutral (Table 1) and is the maximum for N_{FA} , while the other is the maximum for W_{FA} . Tint values varied from -1 to 2.5 and $5Y-280-W_{CIE}$ varied from 7 to 81, all within the CIE bounds.

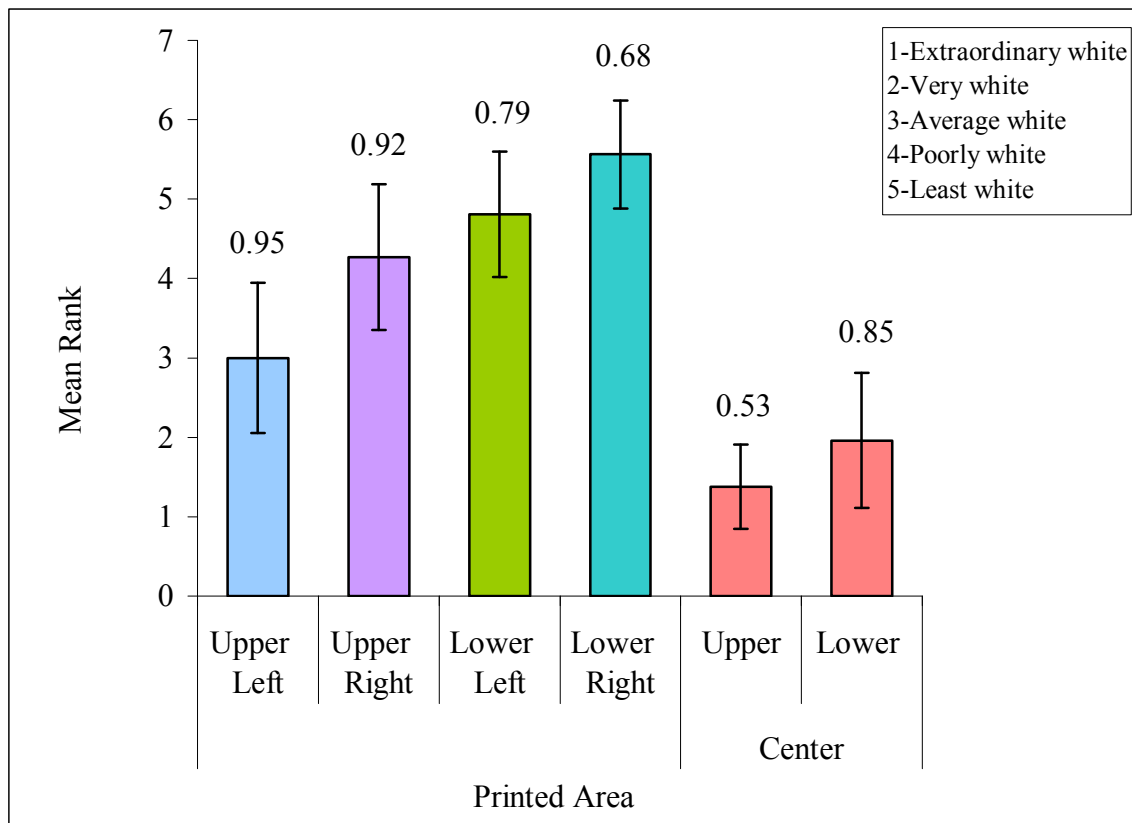


Figure 9. Observers' evaluation for print of Figure 7.

The calculated CIE, Hunter, Ganz and proposed whiteness values, along with TAPPI brightness are given in Figures 10a, b, and c for each printed area of Figure 7 for $D_{65}/10^\circ$, $D_{50}/2^\circ$ and $C/2^\circ$, respectively. As seen in the Figures, the proposed whiteness formulas are in good agreement with the observer evaluation data for each printed area on the sample. The correlation

coefficients, rank correlations and combined correlations for the printed areas (4 corners) are given in Table 4. N_{FA} , W_{FA} and TAPPI brightness showed significantly higher correlations than CIE, Hunter and Ganz whiteness values for the printed areas on the sample. Although the observer evaluations under the daylight behind a window exhibited slightly lower correlations than the observer evaluations under the light booths, the difference is insignificant for these printed samples.

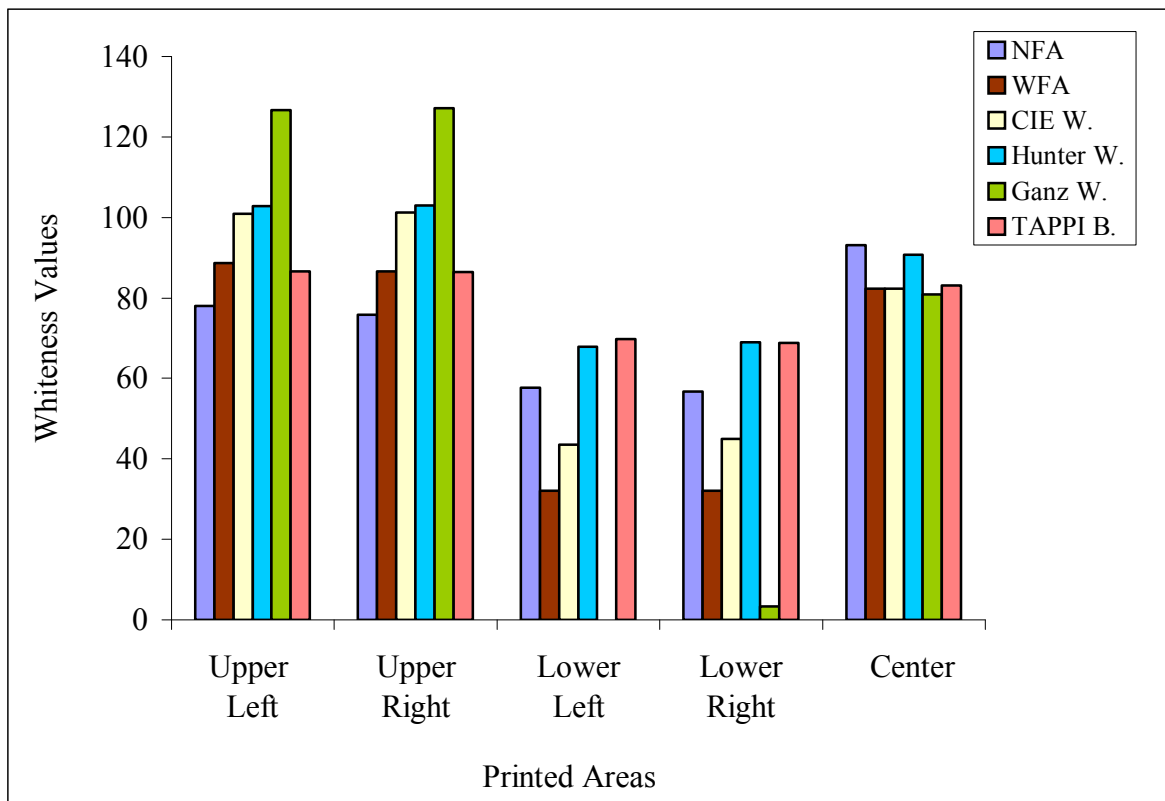


Figure 10a. Whiteness values for $D_{65}10^\circ$.

The correlation coefficients, rank correlations and combined correlations for the printing papers are given in Table 5. Negative values result from increasing whiteness corresponding to a low rank order. All of the formulas except the N_{FA} were in good agreement of the assessments of the printing papers. All of the papers were well within the CIE bounds (4a, b).

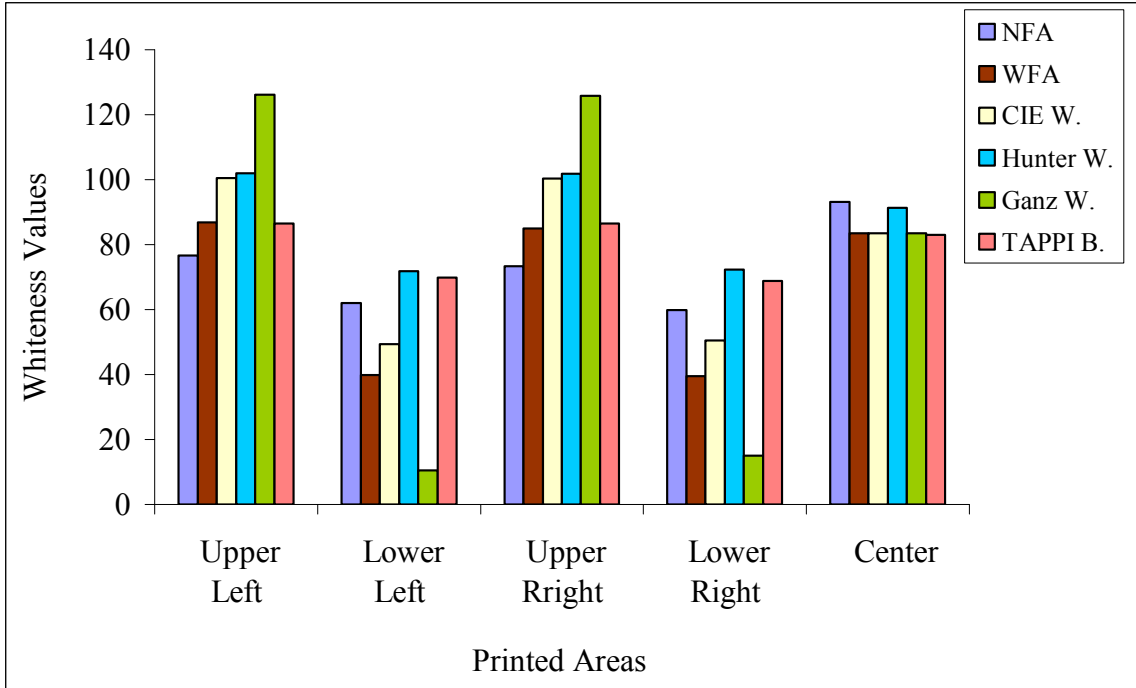


Figure 10b. Whiteness values for D₅₀, 2°.

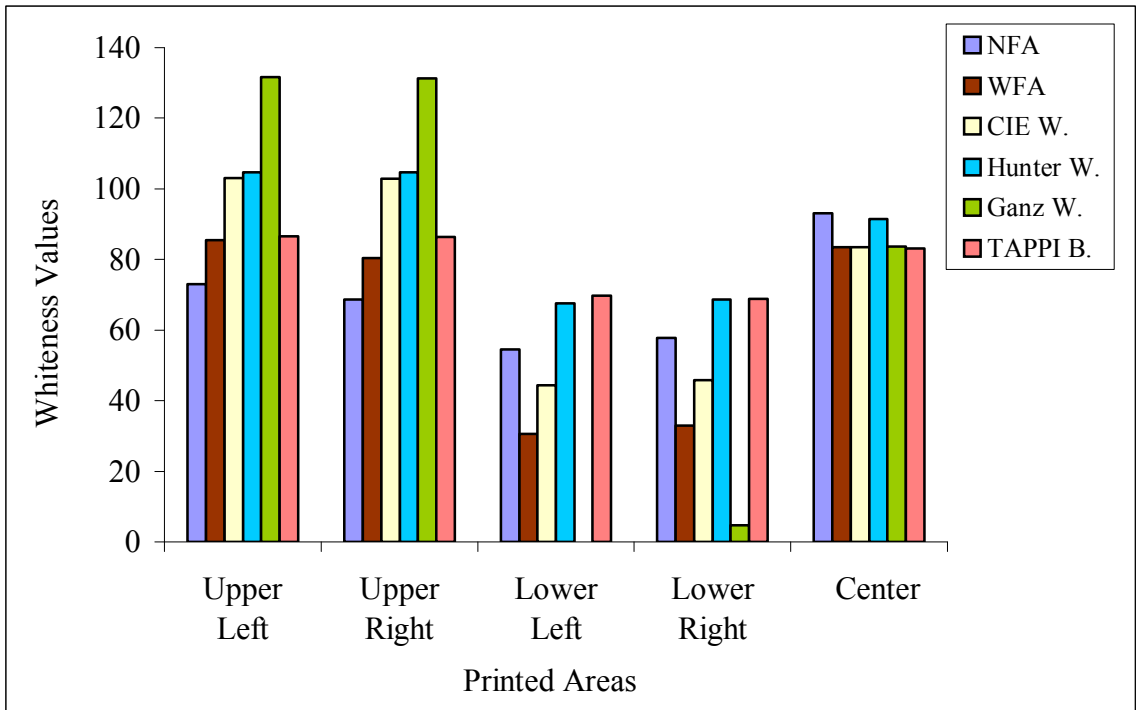


Figure 10c. Whiteness values for C, 2°.

Table 4. Corresponding Correlation Coefficients for the Printed 4 Corners.

	Under Light Booth (25 Obs.)			Under Daylight (23 Obs.)			Combined Obs. Conditions		
	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³
N_{FA}	-0.95	0.94	-0.95	-0.90	0.91	-0.89	-0.93	0.94	-0.93
W_{FA}	-0.87	0.83	-0.87	-0.76	0.77	-0.75	-0.83	0.83	-0.82
CIE	-0.76	0.43	-0.76	-0.64	0.29	-0.62	-0.71	0.43	-0.71
Hunter	-0.75	0.43	-0.75	-0.62	0.29	-0.61	-0.70	0.43	-0.70
TAPPI	-0.84	0.83	-0.84	-0.73	0.77	-0.72	-0.79	0.83	-0.79
GANZ	-0.74	0.43	-0.74	-0.61	0.29	-0.60	-0.68	0.43	-0.68

¹CC: Correlation coefficient

²RCC: Ranking correlation coefficient

³CCC: Combined correlation coefficient

Table 5. Corresponding Correlation Coefficients for the Printing Papers.

	Under Light Booth (25 Obs.)			Under Daylight (23 Obs.)			Combined Obs. Conditions		
	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³
N_{FA}	0.80	-0.50	0.78	0.84	-0.50	0.84	0.82	-0.50	0.81
W_{FA}	-0.99	1.00	-0.99	-0.98	1.00	-0.98	-0.98	1.00	-0.99
CIE	-0.95	1.00	-0.94	-0.97	1.00	-0.97	-0.97	1.00	0.96
Hunter	-0.94	1.00	-0.93	-0.97	1.00	-0.97	-0.96	1.00	-0.95
TAPPI	-0.99	1.00	-0.99	-1.00	1.00	-1.00	-1.00	1.00	-0.99
GANZ	-2.70	1.00	-0.92	-0.96	1.00	-0.96	-0.94	1.00	-0.94

¹CC: Correlation coefficient

²RCC: Ranking correlation coefficient

³CCC: Combined correlation coefficient

The results for the step samples (Tables 2 and 3) were more varied. There appeared to be

some confusion and mix ups of patches that were near to one another colorimetrically. This is not so surprising, because the steps were designed to have $\Delta E \sim 1$. In addition, there was much more variation between the different observers than there was for the patterns based on Figure 7 or the papers. This undoubtedly reflects different observer preferences for samples that have high luminance and are reasonably white by all of the measures.

The correlation coefficients, rank correlations and combined correlations for the silk paper are given in Table 6. The averaged ranked observer data are consistent with two local maxima in whiteness, one near the neutral maximum of N_{FA} and the other near the bluer maximum of W_{FA} . The observer evaluations under the daylight behind a window exhibited significantly lower correlations than the observer evaluations under the light booths. This reduced overall correlation values significantly.

The W_{FA} gives a much better correlation with the observer data than N_{FA} , having correlation coefficients of -0.86 versus -0.22 and rank correlation coefficients of 0.80 versus 0.31 for the light booth evaluation. On the other hand, the W_{FA} and N_{FA} give close correlations with the observer data, having correlation coefficients of -0.59 versus -0.56 and rank correlation coefficients of 0.67 versus 0.46, respectively for the combination of the two observer evaluation conditions. All of the steps are within the CIE inequalities, except for steps 1, 2 and 3, which have tint values slightly greater than 3. However, these would be in bounds if their a^* values were increased slightly (less than .4 in each case). Thus, these steps are visually indistinguishable from ones that are in bounds.

Table 6. Corresponding Correlation Coefficients for the Steps on Silk Paper.

	Under Light Booth (25 Obs.)			Under Daylight (23 Obs.)			Combined Obs. Conditions		
	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³
N_{FA}	-0.22	0.31	-0.18	-0.76	0.70	-0.64	-0.56	0.46	-0.46
W_{FA}	-0.86	0.80	-0.83	-0.14	0.23	-0.11	-0.59	0.67	-0.53
CIE	-0.59	0.35	-0.59	0.35	-0.40	0.33	-0.16	0.04	-0.16
Hunter	-0.57	0.35	-0.57	0.36	-0.40	0.34	-0.14	0.04	-0.15
TAPPI	-0.80	0.68	-0.78	0.04	0.00	0.04	-0.46	0.47	-0.43
GANZ	-0.53	0.35	-0.53	0.41	-0.40	0.38	-0.09	0.04	-0.11

¹CC: Correlation coefficient

²RCC: Ranking correlation coefficient

³CCC: Combined correlation coefficient

The correlation coefficients, rank correlations and combined correlations for the cover paper are given in Table 7. Compared to observation under light booth, correlation coefficients for observations under sunlight behind a window were fairly low. This caused overall calculated correlation coefficients values to fall significantly. It seems that confusion and mix ups of patches that were near to one another colorimetrically were much more apparent for the observation made under sunlight behind a window. The averaged ranked observer data show local maxima in whiteness, near step 4 and step 6, as predicted by our W_{FA} formula. However there is also a local maximum in the observer data at step 1, the bluest printed sample, as would be expected from the CIE, Hunter, and Ganz formulas and from TAPPI brightness.

The user data were even more varied for the cover paper than the silk paper. As a result,

the average rankings, and hence the predicted ranking order, were very sensitive to the weight of averaging. For the observations under booth light, for example, if a rank weighted average is used, W_{FA} has a correlation coefficient of $-.92$, while if the inverse of the average inverse rank is used the correlation coefficient is $-.54$. For TAPPI brightness the correlation coefficient is $-.67$ for rank weighting and $-.78$ for inverse weighting. For the rank weighting, the coefficients were $-.67$ for CIE and $-.66$ for Hunter and Ganz. For inverse weighting, the coefficients were $-.76$ for CIE and Hunter and $-.75$ for Ganz. Because the reliability of the observers is variable, it is hard to know the correct weighting. It doesn't make much difference for the other printed samples, but, as we see, the cover paper samples are very sensitive. A better estimate of the correlations is to take arithmetic averages of the straight, rank and inverse average coefficients. This yields coefficients of $-.77$ for W_{FA} , $-.79$ for CIE, $-.80$ for TAPPI brightness and $-.78$ for Hunter and Ganz. There isn't a great deal of difference between these values and none are very compelling. Clearly, it is not possible to differentiate among the various formulas based on this sample. Additional test patches would need to be constructed to definitively differentiate between the various formulas. The observer data for sunlight through a window are virtually uncorrelated for these steps.

All of the steps are within the CIE bounds, except for steps 1, 2 and 5, which have tint values slightly greater than 3. As with the silk paper, these can be brought into bounds by increasing their a^* values by about $.5$ or less. Because these are visually indistinguishable from patches that are within bounds, the tint deviation is an unlikely cause for the variability of the observer data for the cover paper. In fact, many observers found steps 1 and 2 to be among the whitest samples.

For all of the 41 patches rated by the viewers under booth light, W_{FA} and TAPPI brightness each gave an average correlation coefficient of -0.87, while CIE, Hunter and Ganz showed average correlation coefficients of only -0.77, -0.76, -0.75, respectively. The corresponding averages over straight, rank and inverse weighted averages are -0.83, -0.82, -0.73, -0.72 and -0.70 for W_{FA} , TAPPI, CIE, Hunter and Ganz, respectively. The combination of observations under booth light and under sunlight behind a window, W_{FA} and TAPPI brightness gave average correlation coefficients of -0.71, and -0.65, respectively, while CIE, Hunter, Ganz and N_{FA} showed average correlation coefficients of only -0.52, -0.51, -0.49 and -0.26, respectively. The corresponding averages over straight, rank and inverse weighted averages are -0.65, -0.60, -0.48, -0.46, -0.45 and -0.27 for W_{FA} , TAPPI, CIE, Hunter, Ganz and N_{FA} , respectively. These are summarized in Table 8.

Table 7. Corresponding Correlation Coefficients for the Steps on Cover Paper.

	Under Light Booth (25 Obs.)			Under Daylight (23 Obs.)			Combined Obs. Conditions		
	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³	CC ¹	RCC ²	CCC ³
N_{FA}	0.77	-0.81	0.62	-0.30	0.37	-0.31	0.42	-0.53	0.28
W_{FA}	-0.85	0.80	-0.77	-0.01	0.04	-0.02	-0.65	0.63	-0.54
CIE	-0.93	0.90	-0.79	0.17	-0.17	0.17	-0.61	0.65	-0.46
Hunter	-0.92	0.90	-0.78	0.17	-0.17	0.18	-0.60	0.65	-0.45
TAPPI	-0.94	0.90	-0.80	0.13	-0.17	0.14	-0.64	0.65	-0.49
GANZ	-0.92	0.90	-0.78	0.18	-0.17	0.19	-0.60	0.65	-0.45

¹CC: Correlation coefficient

²RCC: Ranking correlation coefficient

³CCC: Combined correlation coefficient

Table 8. Corresponding Weighted Average Correlation Coefficients.

	Under Light Booth (25 Obs.)		Under Daylight (23 Obs.)		Combined Obs. Conditions	
	AveCC	AveSRIC	AveCC	AveSRIC	AveCC	AveSRIC
W_{FA}	-0.87	-0.83	-0.34	-0.33	-0.71	-0.65
CIE	-0.77	-0.73	-0.09	-0.09	-0.52	-0.48
Hunter	-0.76	-0.72	-0.08	-0.08	-0.51	-0.46
TAPPI	-0.87	-0.82	-0.24	-0.21	-0.65	-0.60
GANZ	-0.75	-0.70	-0.06	-0.06	-0.49	-0.45

¹AveCC: Average correlation coefficient

²AveSRIC: Average straight, rank and inverse correlation

5. CONCLUSIONS

The proposed colorimetric whiteness, N_{FA} and whiteness, W_{FA} , formulas matched well with 48 randomly selected observer assessments for samples printed based on Figure 7. The CIE, Hunter, and Ganz showed significantly worse correlations, while the TAPPI brightness showed a good correlation, though slightly worse than N_{FA} and W_{FA} . However, for samples that are much closer to neutral and the “paper whites”, the results are less definitive. While W_{FA} exhibited the best correlation, TAPPI brightness correlated almost as well. The CIE, Hunter and Ganz formulas correlate much less well than W_{FA} , or TAPPI.

For the observer evaluations, the W_{FA} formula showed a clear improvement over CIE, Hunter and Ganz formulas. The surprising thing is that the TAPPI brightness,⁴³ which is a weighted average of the reflectance factor in the blue region of the visual spectrum with peak wavelength of 457 nm, correlates so well. Because of its dependence on a single region of the

spectrum, it can lead to absurd results. If we consider a hypothetical reflectance spectrum that is the same as the cover paper for wavelengths less than 500 nm and then decreases linearly from 88% at 510 nm to 25% at 730 nm, such a sample would have the same TAPPI brightness as the cover paper! However, its colorimetric values would be $L^*=89.5$, $a^*=-9.4$ and $b^*=-17.9$. This sample is far outside the CIE bounds ($T=21.6$ and $5Y-280-W_{CIE}=-64.2$). W_{FA} for this hypothetical sample is 2.8, consistent with the obvious blue to cyan of this sample. Although some refinements and testing are still in order, the W_{FA} formula is clearly an improvement over the other whiteness assessments. An important advantage is that it doesn't have to rely on a tint formula, since it automatically weights highly tinted samples lower than less tinted samples. In a future publication⁴⁴, we will compare observer observations of whiteness of coated paper samples with the W_{FA} formula.

Correlation coefficients found were the highest for each tested whiteness formulas when observer evaluations were executed under a light booth. Observer evaluations made under daylight behind a glass significantly reduced the correlation coefficients for each whiteness formula investigated. Observer evaluations executed under daylight behind a glass other than a standard light booth added discrepancies to the test results. Still, the reductions in the correlation coefficients were significantly smaller for W_{FA} than for other whiteness formulas tested, where observer evaluations were executed under day light behind a glass.

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LIST OF FIGURES

- Figure 1. Chromaticity diagram
- Figure 2. a. Added Y on chromaticity diagram,
b. Added Y on chromaticity diagram (Looking Down).
- Figure 3. L, a, b color space
- Figure 4. a. Four corner positions at each white point for Y=100 at D50/10.
b. Four corner positions at each white point for Y=100 at D50/10.
- Figure 5. a. Planes in the white region at varying Y values at D50/10.
b. Planes in the white region at varying Y values at D50/10.
- Figure 6. a. CIE equation on L, a*, b* color space.
b. W_{FA} equation on L, a*, b* color space.
c. Comparison of CIE and W_{FA} equation on L, a*, b* color space.
- Figure 7. Pattern printed on Xerox Phaser 8200 printer for viewer observations.
- Figure 8. Coated color samples in the chromaticity diagram for their Y values.
- Figure 9. Observers' evaluation for print of figure 7.
- Figure 10. a. Whiteness values for D_{65} , 10° .
b. Whiteness values for D_{50} , 2° .
c. Whiteness values for C, 2° .