Do TWO of the following three problems.

8. The product topology on $\prod(X_\alpha, T_\alpha)$ has the property that for every space $Y$ and every function

$$f = (f_\alpha) : Y \to \prod X_\alpha,$$

the function $f$ is continuous if and only if each $f_\alpha$ is continuous.

a. Explain why the topology with basis

$$\left\{ \prod U_\alpha \mid U_\alpha \in T_\alpha \right\}$$

is not the product topology.

b. What is a basis for the product topology? Explain why this basis has the correct property.

9. Let $X$ be a compact space and $Y$ a metric space. Prove that a sequence of continuous functions $(f_n : X \to Y)$ converges to a function $f : X \to Y$ in the compact-open topology if and only if the sequence $(f_n)$ converges uniformly to $f$.

10. Let $X_1 \subseteq X_2 \subseteq X_3 \subseteq \cdots$ be a sequence of normal spaces, where each $X_i$ is a closed subset of $X_{i+1}$. Let $X = \bigcup X_i$. Define a topology on $X$, by defining a subset $U$ of $X$ to be open if $U \cap X_i$ is open in $X_i$ for each $i$.

Prove that $X$ is a normal space. [Hint: Use the Tietze Extension Theorem and Urysohn’s Lemma.]