WMU ANALYSIS COMPREHENSIVE EXAMINATION

Show all work! March 4, 2005

1)

(a) State and prove a general theorem about the differentiability of

$$F(y) = \int_0^\infty f(x, y)\, dx.$$

(b) Show that

$$F(y) = \int_\pi^\infty \frac{e^{-xy}\sin(x)}{x}\, dx$$

is differentiable On $(0, \infty)$ and compute $F'(y)$.

2)

(a) State the Riesz Representation Theorem for L^p.

(b) Show that the representation from part (a) is unique.

(c) How does the Riesz Representation Theorem for L^p relate to linear functionals on the space of continuous functions?

3) Define the sum of two subsets A and B of $\mathbb{R}$ by $A + B = \{x + y \mid x \in A \text{ and } y \in B\}$. Prove or disprove: if A and B are Lebesgue measureable, then so is $A + B$.

4) Show that the set of continuous functions on an interval $[a, b] \subset \mathbb{R}$ are not dense in $L^\infty[a, b]$.

5) Suppose that f is in $L^1[a, b]$ and ϕ is $C^\infty[a, b]$. Define the convolution of f and ϕ to be $\text{conv}\,(f, \phi) = \frac{1}{b-a}\int_a^b f(x-h)phi(h)\, dh$. Show that conv (f, ϕ) is $C^\infty[a, b]$.