Do TWO of the following three problems.

8. The product topology on $\prod(X_\alpha, \mathcal{T}_\alpha)$ has the property that for every space Y and every function

$$f = (f_\alpha) : Y \longrightarrow \prod X_\alpha,$$

the function f is continuous if and only if each f_α is continuous.

a. Explain why the topology with basis

$$\left\{ \prod U_\alpha \,\middle|\, U_\alpha \in \mathcal{T}_\alpha \right\}$$

is not the product topology.

b. What is a basis for the product topology? Explain why this basis has the correct property.

9. Let X be a compact space and Y a metric space. Prove that a sequence of continuous functions $\langle f_n : X \longrightarrow Y \rangle$ converges to a function $f : X \longrightarrow Y$ in the compact-open topology if and only if the sequence $\langle f_n \rangle$ converges uniformly to f.

10. Let $X_1 \subseteq X_2 \subseteq X_3 \subseteq \cdots$ be a sequence of normal spaces, where each X_i is a closed subset of X_{i+1}. Let $X = \bigcup X_i$. Define a topology on X, by defining a subset U of X to be open if $U \cap X_i$ is open in X_i for each i.

Prove that X is a normal space. [*Hint:* Use the Tietze Extension Theorem and Urysohn's Lemma.]