DOCTORAL ENTRANCE EXAMINATION
GENERAL TOPOLOGY
JANUARY 18, 1997

Please use your own paper.

Do FIVE of the following seven problems.

1. Let X be a compact space and Y a Hausdorff space. Show that a continuous bijection $f : X \longrightarrow Y$ is a homeomorphism.

2. Let X be a compact space and $f : X \longrightarrow \mathbb{R}$ be a continuous function. Show that the image of f has both an absolute maximum and absolute minimum.

3. Show that the image of a continuous function from a connected space is connected.

4. Let $X = \{1, 2, 3\}$ be a set and $Y = \{a, b\}$ be a space with the discrete topology. Define $f : X \longrightarrow Y$ by $f(1) = a$, $f(2) = a$, and $f(3) = b$. Describe the coarsest topology on X where f is a continuous function.

5. Let X be a finite set. Describe all of the Hausdorff topologies on X.

6. Let X be a topological space with the property that whenever C and D are disjoint closed subsets of X, there are disjoint open subsets U and V such that $C \subseteq U$ and $D \subseteq V$. Does it follow that X is a Hausdorff space? Either prove your answer or give a counterexample.

7. Let $A = \{\frac{1}{n} \mid n \in \mathbb{N}\}$ and $\mathcal{B} = \{(a, b) \mid a < b\} \cup \{(a, b) - A \mid a < b\}$. Prove that $\mathcal{B}$ is a basis for a topology $\mathcal{T}$ on $\mathbb{R}$. Is $\mathcal{T}$ the usual topology on $\mathbb{R}$?