DOCTORAL ENTRANCE EXAMINATION
GENERAL TOPOLOGY
MAY 10, 1997

Please use your own paper.

Do EIGHT of the following ten problems.

1. Let $\mathcal{T}_1, \mathcal{T}_2$ be two topologies on a set X. Show that $\mathcal{T}_1 \cap \mathcal{T}_2$ is a topology, but give an example to show that $\mathcal{T}_1 \cup \mathcal{T}_2$ need not be.

2. If A and B are subsets of a topological space X with $A \subseteq B$, then every limit point of A is a limit point of B (i.e., $A' \subseteq B'$).

3. Can a topology be determined by specifying the closed subsets? If so, what axioms (or properties) should a collection of closed subsets have?

4. a. Define the product topology on $\prod_{\lambda \in \Lambda} X_\lambda$.
   b. Explain why the product topology is preferable over either the box or uniform topology.
   c. Discuss some properties of the product topology.

5. Show $\mathbb{R}$ is not homeomorphic to $\mathbb{R}^2$.

6. Show that a function $f : X \to Y$ is continuous if and only if, for every subset A of X, $f(\overline{A}) \subseteq \overline{f(A)}$.

7. Prove that the identity function $i : (X, \mathcal{T}) \to (X, \mathcal{T}^*)$ is continuous if and only if $\mathcal{T}$ is finer than $\mathcal{T}^*$ (i.e., $\mathcal{T}^* \subseteq \mathcal{T}$).