

8. Show that every compact subset of a metric space is bounded in that metric and is closed. Find a metric space in which not every closed and bounded subset is compact.

9. If Y is a Hausdorff space, the function space of continuous functions from X to Y , $\mathcal{C}(X, Y)$, with the compact-open topology is Hausdorff.

10. Define an equivalence relation on \mathbb{R}^2 by

$$(a, b) \sim (c, d), \quad \text{if } a + b^2 = c + d^2.$$

The equivalence classes are parabolas of the form $x + y^2 = k$ where k is a constant. Let X be the collection of equivalence classes with the quotient topology. Show X is homeomorphic to a familiar space.