Autonomous Eco-Driving with Traffic Light and Lead Vehicle Constraints: An Application of Best Constrained Interpolation


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Abstract: Eco-Driving is a critical technology for improving automotive transportation efficiency. It is achieved by modifying the driving trajectory over a particular route to minimize required propulsion energy. Eco-Driving can be approached as an optimal control problem subject to driving constraints such as traffic lights and positions of other vehicles. In this paper we demonstrate the connection between Eco-Driving and best interpolation in the strip which is a problem in approximation theory and optimal control. By exploiting this connection, we are able to generate optimal Eco-Driving trajectories that can be driven with an autonomous system and evaluate them using conventional, hybrid electric, and fully electric vehicle models from FASTSim software. Our results quantify the energy efficiency improvements that can be achieved with the proposed approach.

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Keywords: Eco-Driving, constrained control, industrial applications of optimal control, autonomous vehicles, electric vehicle, intelligent transportation systems.

1. INTRODUCTION

In the U.S., the National Highway Traffic Safety Administration’s (NHTSA) Corporate Average fuel economy (CAFE) requirement is gradually raised on an annual basis to reduce the adverse consequences of automotive transportation (NHTSA, 2020). These adverse consequences include economic costs for petroleum importation and vulnerability to geopolitical stability (Greene and Liu, 2015), accelerated global warming due to large amounts of greenhouse gas emissions (U.S. EIA, 2020), and adverse effects on human health due to air pollution (Anenberg et al., 2019; U.S. EPA, 2015). To date, the CAFE policy has been shown to be effective in promoting energy efficiency technology in modern commercially-available vehicles (Zielinski et al., 2019).

An effective method of achieving large energy efficiency improvements in vehicles is to implement Eco-Driving. Eco-Driving is the realization of a more energy efficient drive cycle for the same route and is typically achieved by eliminating full stops, maintaining a constant speed, limiting acceleration, and smoothing the velocity profile (Gondar et al., 2012; Michel et al., 2016; Huang et al., 2018). Implementing energy efficient driver behavior reduces the vehicle propulsion energy required to drive the route, though it typically results in longer travel time and has historically lower user acceptance by human drivers (Gondar et al., 2012). However, human driver resistance may be reduced when combined with driving automation technologies.

Automation technologies, such as advanced driving assistance systems (ADAS) and autonomous vehicles have high user acceptance (Rödel et al., 2014), and it is likely that users can look past the increased travel time from Eco-Driving in exchange for not having to drive. Recent research has shown that when implementing a heuristic set of goals such as removing stops, traveling at an energy efficient speed (in general, this could be a higher or lower overall speed), and limiting acceleration and deceleration magnitudes, it is possible to achieve Fuel Economy (FE) improvements of approximately 10% for modern vehicles and 30% for fully autonomous vehicles (Michel et al., 2016). Also, when Eco-driving is applied to electrified vehicles the result is longer battery-life and slower battery degradation (Mohan et al., 2020). Additionally, vehicles without Eco-Driving technology can still improve their energy efficiency if they follow an Eco-driving equipped lead vehicle (U.S. DOE, 2020).
The problem of maximizing the energy efficiency improvements from Eco-Driving can be formulated as an optimal control problem if a prediction of driving conditions along the route is available. These predictions are informed by vehicle sensors and vehicle to vehicle (V2V) and vehicle to infrastructure (V2I) communication technologies. As an example of a typical Eco-Driving study, predictions of traffic light signal phase and timing (a V2I technology) were used to change driving behavior and demonstrated a FE improvement of 12-14% (Mandava et al., 2009). Another study showed that using V2I-enabled traffic lights decreases the energy consumed in Eco-Driving (U.S. DOE, 2020). Other approaches include the optimization of speed and power distribution between the motor and engine in a connected HEV subject to A/C thermal load (Amini et al., 2019), imposing constraints on vehicles that are queued for a green light (Yang et al., 2019; Iliev et al., 2019), and using information from the vehicle in front and traffic light data to avoid red lights (Bae et al., 2009). Additionally, applications of Model Predictive Control (MPC) and Dynamic Programming (DP) have been considered in a lead vehicle following scenario (Prakash et al., 2017).

Our approach here is unique compared to past efforts. In this paper, we demonstrate a connection between Eco-Driving and a problem in approximation theory and optimal control (Dontchev, 1993; Dontchev and Kolmanovsky, 1995, 1996) which is referred to as “best interpolation in a strip”. By exploiting this connection, we show that it is possible to quickly generate optimal Eco-Driving trajectories for real-world driving scenarios with traffic lights and lead vehicle constraints. As expected, these derived optimal trajectories show significant energy efficiency improvements when used in real-world validated models and compared to EPA reported FE. Other connections between control theory and spline interpolation are illustrated in (Egerstedt and Martin, 2009).

2. METHODS

We consider a problem of generating an Eco-Driving trajectory subject to real world driving constraints of traffic lights and avoiding collisions with a lead vehicle. Because precise vehicle control is required, this has primary applications in autonomous or assisted control rather than human driver control.

2.1 Problem Statement and Assumptions

Several assumptions are made. Specifically we assume that we are able to maintain precise control of the ego vehicle velocity on a second-by-second basis, which would be enabled by autonomous or assisted vehicle control. Previous research into Eco-Driving suggests that the Eco-Driving can be achieved without high-fidelity models of vehicle fuel consumption; instead it is sufficient to minimize acceleration (Prakash et al., 2017, 2016, 2019) and, whenever possible, pass through the traffic lights when they are green to avoid braking, stopping and idling. This approach is adopted in this paper as well.

We further assume that the ego vehicle is following a lead vehicle which is represented here with a simple piecewise linear distance vs time function. We consider asynchronous traffic light signal phase and timing (SPaT) along a straight route during which the vehicle will stop for red lights. Each of these assumptions is summarized as follows:

1. Vehicle trajectory is to be specified as a function of time
2. Minimizing the integral of the square of the acceleration over time increases energy efficiency
3. Piecewise linear lead vehicle constraint
4. Asynchronous SPaT along a straight drive cycle

These assumptions lead to an optimal control formulation which is also linked to a problem in approximation theory: the best interpolation in a strip.

2.2 Optimal Control Problem Formulation

Fig. 1 shows a visualization of the distance vs time problem geometry with SPaT data of three traffic lights, forecasted lead vehicle trajectory \((d(t), \text{blue line})\) and the trajectory window just before the traffic light turns red \((e(t), \text{pink line})\). It can be seen that the lead vehicle stopped at the first red light phase of the first traffic light, then stopped for almost the entire duration of the second red light phase of the second traffic light, and, finally, encountered the third traffic light in a green phase and thus drove through the intersection.

Consider now the ego vehicle’s position trajectory vs time, \(f(t)\), which has to connect the waypoints which are designated by black circles in Fig. 1. These waypoints lie somewhere within the corresponding gates delineated by orange double ended arrows. These gate locations are set based on changes in boundary conditions (e.g. the lead vehicle transitions from zero velocity to some constant...
velocity). The ego vehicle velocities at each waypoint are indicated by black lines. The waypoint locations, the vehicle velocities at the waypoints, and the trajectory “pieces” between the waypoints are determined by the theory of best interpolation in a strip. Note that the lower constraint, $e(t)$, is determined based on trailing red light SPaT phases (in other words, the ego vehicle must pass through each intersection before the light turns red).

By assigning the orange markers as “gates”, we can now designate the interval between gates as the “strip”. Shown in the inset dashed green circle in Fig. 1 is a strip where the left boundary (the start time of the strip) is designated as $t_i$ and the right boundary (the end time of the strip) is designated as $t_{i+1}$. The ego vehicle position at certain time instant $f(t_i)$, where $i$ is the discrete time step index, is bounded above by $d(t_i)$ and below by $e(t_i)$. Overall, the ego vehicle trajectory has to adhere to the following constraints:

1. $f(t) \leq d(t)$, where $d(t)$ is informed by the forecasted lead vehicle position offset by safety margin;
2. $f(t) \geq e(t)$, where $e(t)$ is informed by the transition of the traffic signal phase to red.

Note that constraints $e(t) \leq f(t) \leq d(t)$ form a strip that has two gates in distance vs. time that the ego vehicle must pass through similar to the way a slalom skier passes through gates during a competition. However, unlike a slalom skier, who pursues a minimum time trajectory, we seek to minimize the $L_2$ norm of the ego vehicle acceleration trajectory which in turn improves energy efficiency.

The constraints $e(t)$ and $d(t)$ are assumed to be piecewise linear; they are linear functions of time in each time interval $[t_i, t_{i+1}]$, where the time intervals are fixed. There are a few caveats regarding this. Firstly, for real-world lead vehicles, this corresponds to an approximated representation. The theory of best interpolation in a strip does allow for piecewise cubic constraints which could be incorporated in future work. Secondly, the continuity of $e(t)$ is enforced by not allowing infinite slope in the $e(t)$ function as can be seen in Fig. 1. Lastly, it should also be mentioned that realistically, the $d(t)$ constraint should include a distance margin from a lead vehicle.

To demonstrate the concept of this paper, we will use the routes shown in Figs. 2 and 3 with their associated real-world red light locations and speed limits. We will designate the Fig. 2 route as Kalamazoo Arterial (KA) and the Fig. 3 route as Kalamazoo Downtown (KD).

For these routes, real world vehicle velocity was recorded along two different roads in Kalamazoo, MI: one along Stadium drive to serve as the arterial route and one along Kalamazoo Mall to serve as the downtown route. A consistent SPaT cycle of 120 seconds was used for this demonstration since it represents a broad use case (Koonce et al., 2008; NACTO, 2013). This SPaT data then informs specific piecewise linear upper and lower constraints for the optimal control problem. We note that the lead vehicle constraints in this paper are defined by speed limits and converted to piecewise linear functions. Based on results in (Dontchev, 1993; Dontchev and Kolmanovsky, 1995, 1996), piecewise cubic constraints can also be handled, but this is left to future work.

![Fig. 2. Details of the Kalamazoo Arterial route which is used to collect lead vehicle and SPaT data constraints.](image1)

![Fig. 3. Details of the Kalamazoo Downtown route which is used to collect lead vehicle and SPaT data constraints.](image2)

In the above and subsequent developments we assumed that there is either no vehicle trailing the ego vehicle, or such a trailing vehicle does not influence the ego vehicle trajectory. In principle, this assumption could be relaxed, e.g., the lower constraint on the ego vehicle trajectory can be additionally informed by a trailing vehicle trajectory in order to minimize disruption to its motion.

### 2.3 Optimal Control Solution

The eco-driving problem set-up exploits the connection with the problem of best interpolation in a strip (Dontchev, 1993; Dontchev and Kolmanovsky, 1995, 1996). In particular, the planning of vehicle distance trajectory versus time, $f(t)$, $t_0 \leq t \leq t_N$, can be posed as the following problem:

Given time instants $t_0 < t_1 \cdots < t_N$, initial and final slopes $s_0$ and $s_N$, lower and upper bounding functions $e$ and $d$ satisfying $e(t) < d(t)$ for all $t \in [t_0, t_N]$ which are continuous on $[t_0, t_N]$, and linear on $[t_i, t_{i+1}]$, determine the function $f$ that satisfies the following optimization problem:
Minimize \( f''(t) \) \( L^2[0, t_N] \)
subject to
\[
e(t) \leq f(t) \leq d(t), \quad t_0 \leq t \leq t_N,
\]
The computational approach to finding such a function \( f \)
then decomposes into an outer and an inner optimization
problems. The outer loop problem is to minimize the function
\[
\phi(y, s) = \sum_{i=0}^{N-1} \phi_i(y_i, s_i, y_{i+1}, s_{i+1}),
\]
where \( y = (y_0, \ldots, y_N) \) and \( s = (s_0, \ldots, s_N) \) are vectors of
function values and slopes at \( t_i \) where \( i = 0, \ldots, N \),
and \( \phi_i(y_i, s_i, y_{i+1}, s_{i+1}), i = 0, \ldots, N - 1 \) are computed
by solving \( N \) independent inner optimization problems:
Minimize \( f(c) \)
subject to
\[
\begin{align*}
f(t_i) &= y_i, \quad f(t_{i+1}) = y_{i+1} \\
f(t_i) &= s_i, \quad f(t_{i+1}) = s_{i+1} \\
e(t) &\leq f(t) \leq d(t), \quad t_i \leq t \leq t_{i+1}.
\end{align*}
\]
The function \( \phi \) can be shown to be a convex and coercive
function of the arguments \( y \) and \( s \), and it has a unique
minimum (Dontchev, 1993; Dontchev and Kolmanovsky,
1995, 1996). The solution to the inner loop optimization
problem (3) is a \( C^2 \) cubic spline that is one of the following
types determined by its interactions with the lower and
upper constraining functions \( e(t) \) and \( d(t), t_i \leq t \leq t_{i+1} \):
Case 1: Constraints are inactive;
Case 2: Single touching point on the lower constraint \( e \);
Case 3: Single touching point on the upper constraint \( d \);
Case 4: Single subarc on the lower constraint \( e \);
Case 5: Single subarc on the upper constraint \( d \);
Case 6: Touching pair with the first touching point on
the lower constraint \( e \) and second touching point on the
upper constraint \( d \);
Case 7: Touching pair with the first touching point on
the upper constraint \( d \) and second touching point on the
upper constraint \( e \).

Note that when \( y_i \) and \( s_i \) are given, the optimal trajectory
of (3) is unique and corresponds to one of the cases 1-7.
In order to find this optimal trajectory we evaluate the cost
for each of the cases for which a feasible solution exists and
select the case corresponding to the minimal value. In that
sense problem (3) is solved as a combinatorial optimization
problem.

As an example, the candidate solution in Case 4 (a single
subarc on the lower constraint) is of the form
\[
f(t) = \begin{cases} 
\sum_{i=0}^{\min(2, \bar{t} - t_i)} \frac{k_1(\bar{t} - t_i)^3 + k_2(\bar{t} - t_i)^2}{3} + k_3(\bar{t} - t_i) + k_4, & t \in [t_i, \bar{t}], \\
e(t), & t \in [\bar{t}, \bar{t}_2], \\
\sum_{i=0}^{\min(2, t_N - t_i)} \frac{m_1(-\bar{t} + t_i)^3 + m_2(-\bar{t} + t_i)^2}{3} + m_3(-\bar{t} + t_i) + m_4, & t \in [\bar{t}_2, t_{N-1}],
\end{cases}
\]
where the subarc on the lower constraint occurs for \( t \in [\bar{t}_1, \bar{t}_2] \). In this case all coefficients \( k_1, k_2, k_3, k_4, m_1, m_2, m_3, m_4 \) and parameters \( \bar{t}_1 \) and \( \bar{t}_2 \) can be computed by solving linear algebraic equations only. The function (2)
is computed by adding up the minimum costs from each interval.

The requirement that \( e \) and \( d \) are piecewise linear can be
relaxed to \( e \) and \( d \) being piecewise cubic but the
calculations become more involved. While an alternative
brute force approach to (1) involving time discretization
and conversion to a quadratic program is also possible, the
disadvantages of that route include higher dimensionality
of the optimization problem and the potential for “inter-
sample” constraint violations (e.g., collisions with lead
vehicle or violation of traffic light rules in extreme cases).

2.4 FE Evaluation

This optimal solution technique is implemented in Matlab
and is used to compute the optimal Eco-Driving control
for the Kalamazoo Arterial and the Kalamazoo Downtown
drive cycles. Once the optimal trajectory of the ego vehicle
has been determined, the FE is evaluated using established
and real-world validated vehicle modeling software. For
this study we chose to use the FASTSim software (Brooker
et al., 2015) which is freely available from the National
Renewable Energy Laboratory (Brooker et al.).

We are interested in quantifying the results of this techni-
que for several different types of vehicle architecture
including Hybrid Electric Vehicles (HEVs), Conventional
Vehicles (CVs), and Electric Vehicles (EVs). To meet this
criteria, we chose a 2016 Toyota Highlander Hybrid, 2016
Toyota Camry 4 cylinder, and 2016 Tesla Model S60. We have
used the Python version of FASTSim for greater flexibility.
It uses the drive cycle as an input and provides
FE as an output.

The optimal trajectory result will be subsequently shown
in distance (mi) versus time (min). In order to apply
FASTSim as our validation tool, the second-by-second
vehicle velocity time history was inferred from the optimal
position trajectory. The road grade was set to zero.

3. RESULTS

The results of this study are the determined upper and
lower constraints, the derived Eco-Driving drive cycle, and
the associated FE for the three vehicle models for the
Kalamazoo Arterial and the Kalamazoo Downtown routes.

3.1 Optimal Control Solution

The constraints and Eco-Driving drive cycles for the two
routes are shown in Figs. 4 and 5. The lower constraint
(red) is the time window just before the traffic turns red
and the upper constraint (blue) is the lead vehicle. The
optimized trajectory is the green spline which in each
time interval, can fall into the seven cases mentioned in section
2.3. Note that Figs. 4 and 5 show two different drive cycles
for each route. This is because the initial condition of the
ego vehicle route is somewhat arbitrary. For both routes,
drive cycle 1 (subfigure a) has an initial slope (velocity of
the ego vehicle) determined by the optimizer, where drive
cycle 2 (subfigure b) has an initial slope chosen to be half
the slope of the upper constraint (lead vehicle velocity).
This choice is meant to provide distinct results and insights
into the optimal solution.
Fig. 4. Kalamazoo Arterial route results for the Eco-Drivering solution with an initial velocity chosen by the optimizer, drive cycle KA1 (a), and an initial velocity assigned as half of the speed limit, drive cycle KA2 (b).

Drive cycles KA1 and KA2 cover a total distance of 3.31 miles in 7.5 minutes. Fig. 4a shows KA1 where the initial slope was chosen by the optimizer. The KA1 trajectory only uses two of the seven possible cases for the optimal solution: case 1 and case 3. The single touching point is located towards the end of the drive cycle at $t = 6.81$ and 5.85 minutes which is the only additional gate other than the 12 fixed gates located at $t = 0, 0.96, 1.96, 2.97, 3.4, 3.96, 4.4, 5.4, 5.8, 6.8, 7,$ and 7.44 minutes. The KA2 trajectory also only shows case 1 and case 3. The touching pair is located towards the end of the drive cycle at $t = 7$ and 6.84 minutes which is the only 2 additional gates other than the 12 fixed gates. Comparing the drive cycle 1 and 2, both trajectories were approximately similar.

Drive cycles KD1 and KD2 cover a total distance of 0.9 miles in 5.9 minutes. Overall this route provides tighter upper and lower constraints for the vehicles trajectory to follow. KD1 again only shows case 1 and case 3. The single touching point is located towards the end of the drive cycle at $t = 4.4$ and 4.6 minutes which is the only additional gate other than the 14 fixed gates located at $t = 0, 0.2, 0.5, 1.3, 1.5, 1.9, 2.6, 2.9, 3, 3.6, 4.1, 4.4, 4.6,$ and 5.9 minutes. The KD2 trajectory slightly exceeds the speed limit to make the stop light at $t = 1.9$ minutes. KD2 also has three cases: case 1, case 2, and case 3. The touching pair is located towards the end of the drive cycle at $t = 1.3$ and 4.6 minutes which is the only 2 additional gates other than the 14 fixed gates.

Fig. 5. Kalamazoo Downtown route results for the Eco-Drivering solution with an initial velocity chosen by the optimizer, drive cycle KD1 (a), and an initial velocity assigned as half of the speed limit, drive cycle KD2 (b).

3.2 FE Results

MPGe (miles per gallon gasoline equivalent) and MPG are calculated for all derived drive cycles according to the standard (SAE, 2010). Each result is then contrasted with EPA FE measurements for city driving (U.S. EPA, 2021) in Table 1. Note that the results vary widely depending on the vehicle model used.

Table 1. Energy efficiency (i.e. FE) of all drive cycles for an HEV, EV, and CV.

<table>
<thead>
<tr>
<th>Drive Cycle</th>
<th>2016 Toyota Highlander Hybrid (HEV) MPG</th>
<th>2016 Tesla Model S60 (EV) MPGe</th>
<th>2016 Toyota Camry 4cyl (CV) MPG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Drive-cycle 1</td>
<td>65.5</td>
<td>220.5</td>
<td>48.6</td>
</tr>
<tr>
<td>Main Drive-cycle 2</td>
<td>65.8</td>
<td>218.5</td>
<td>45.5</td>
</tr>
<tr>
<td>Arterial Drive-cycle 1</td>
<td>41.4</td>
<td>243.7</td>
<td>30.6</td>
</tr>
<tr>
<td>Arterial Drive-cycle 2</td>
<td>45.0</td>
<td>238.6</td>
<td>30.0</td>
</tr>
<tr>
<td>Downtown Drive-cycle 1</td>
<td>41.4</td>
<td>243.7</td>
<td>30.6</td>
</tr>
<tr>
<td>Downtown Drive-cycle 2</td>
<td>45.0</td>
<td>238.6</td>
<td>30.0</td>
</tr>
<tr>
<td>EPA FE (city)</td>
<td>27</td>
<td>98</td>
<td>28</td>
</tr>
</tbody>
</table>

Table 1 shows a significant energy efficiency increase for all vehicles in both routes when compared to EPA city estimates. Additionally, there is a significant difference in the energy efficiency improvements for the arterial roads vs. downtown. This is because the main-arterial roads are known to have a better FE since they have greater distances between stop lights and their speed limit is
higher. Downtown routes have closely spaced traffic lights with lots of acceleration and deceleration. Both the Toyota Camry and the Tesla Model S show FE increases in drive cycle 1 over drive cycle 2 in both routes. The Toyota Highlander Hybrid on the other hand shows a significant increase in drive cycle 2 over drive cycle 1 in both routes.

4. CONCLUSION

This paper proposed an approach for increasing energy efficiency of eco-driving vehicles based on best interpolation in a strip. Two real-world routes were chosen: Kalamazoo Arterial and Kalamazoo Downtown. These routes were converted to distance over time constraints for an optimal control problem to be solvable using best interpolation in a strip. The Eco-Driving solution was used as an input to the real-world validated FASTSim models that were used to simulate the performance of an HEV, CV, and EV. Results show a significant improvement compared to EPA reported FE for city driving.

Eco-Driving is an important technology to realize sustainable transportation and the method presented here can be used in all levels of vehicle automation provided surrounding vehicle and SPaT data is available. Future work includes measuring human driver FE of a vehicle along this route, measuring Eco-Driving automated FE of a vehicle along this route, expanding the optimization problem formulation to allow for cubic constraints, and using real-world measured SPaT data specific to these routes.

REFERENCES

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