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Chapter 6 Rules for Exponents and the Reasons for Them

6.1 INTEGER POWERS AND THE EXPONENT RULES

Repeated addition can be expressed as a product. For example,

$$\underbrace{2+2+2+2+2}_{5 \text{ terms in sum}} = 5 \times 2.$$

Similarly, repeated multiplication can be expressed as a power. For example,

$$\underbrace{2 \times 2 \times 2 \times 2 \times 2}_{5 \text{ factors in product}} = 2^5.$$

Here, 2 is called the *base* and 5 is called the *exponent*. Notice that 2^5 is not the same as 5^2, because $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$ but $5^2 = 5 \times 5 = 25$.

In general, if a is any number and n is a positive integer, then we define

$$a^n = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n \text{ factors}}.$$

Notice that $a^1 = a$, because here we have only 1 factor of a. For example, $5^1 = 5$. We call a^2 the *square* of a and a^3 the *cube* of a.

Multiplying and Dividing Powers with the Same Base

When we multiply powers with the same base, we can add the exponents to get a more compact form. For example, $5^2 \cdot 5^3 = (5 \cdot 5) \cdot (5 \cdot 5 \cdot 5) = 5^{2+3} = 5^5$. In general,

$$a^n \cdot a^m = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{\text{n factors}} \cdot \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{m \text{ factors}} = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n+m \text{ factors}} = a^{n+m}.$$

Thus,

$$a^n \cdot a^m = a^{n+m}.$$

Example 1

Write with a single exponent:

(a) $q^5 \cdot q^7$

(b) $6^2 \cdot 6^3$

(c) $2^n \cdot 2^m$

(d) $3^n \cdot 3^4$

(e) $(x + y)^2(x + y)^3$.

Solution

Using the rule $a^n \cdot a^m = a^{n+m}$ we have

(a) $q^5 \cdot q^7 = q^{5+7} = q^{12}$

(b) $6^2 \cdot 6^3 = 6^{2+3} = 6^5$

(c) $2^n \cdot 2^m = 2^{n+m}$

(d) $3^n \cdot 3^4 = 3^{n+4}$

(e) $(x + y)^2(x + y)^3 = (x + y)^{2+3} = (x + y)^5$.

Just as we applied the distributive law from left to right as well as from right to left, we can use the rule $a^n \cdot a^m = a^{n+m}$ written from right to left as $a^{n+m} = a^n \cdot a^m$.

Example 2

Write as a product:

(a) 5^{2+a}

(b) x^{r+4}

(c) y^{t+c}

(d) $(z + 2)^{z+2}$.

Solution

Using the rule $a^{n+m} = a^n \cdot a^m$ we have

(a) $5^{2+a} = 5^2 \cdot 5^a = 25 \cdot 5^a$

(b) $x^{r+4} = x^r \cdot x^4$

(c) $y^{t+c} = y^t \cdot y^c$

(d) $(z + 2)^{z+2} = (z + 2)^z \cdot (z + 2)^2$.

When we divide powers with a common base, we subtract the exponents. For example, when we divide 5^6 by 5^2, we get

$$\frac{5^6}{5^2} = \frac{\overbrace{5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5}^{\text{6 factors of 5}}}{\underbrace{5 \cdot 5}_{\text{2 factors of 5}}} = \frac{\overbrace{\cancel{5} \cdot \cancel{5}}^{\text{2 factors of 5 cancel}} \cdot \overbrace{5 \cdot 5 \cdot 5 \cdot 5}^{\text{6 − 2 = 4 factors of 5 are left after canceling}}}{\underbrace{\cancel{5} \cdot \cancel{5}}_{\text{2 factors of 5 cancel}}} = \underbrace{5 \cdot 5 \cdot 5 \cdot 5}_{\text{6−2=4 factors}} = 5^{6-2} = 5^4.$$

More generally, if $n > m$,

$$\frac{a^n}{a^m} = \frac{\overbrace{a \cdot a \cdot a \cdot a \cdot \cdot \cdot a}^{n \text{ factors of } a}}{\underbrace{a \cdot a \cdot \cdot \cdot a}_{m \text{ factors of } a}} = \frac{\overbrace{\cancel{a} \cdot \cancel{a} \cdot \cdot \cdot \cancel{a}}^{\substack{m \text{ factors} \\ \text{of } a \text{ cancel}}} \cdot \overbrace{a \cdot \cdot \cdot a}^{\substack{n-m \text{ factors} \\ \text{of } a \text{ are left} \\ \text{after canceling}}}}{\underbrace{\cancel{a} \cdot \cancel{a} \cdot \cdot \cdot \cancel{a}}_{\substack{m \text{ factors} \\ \text{of } a \text{ cancel}}}} = \underbrace{a \cdot a \cdot a \cdot \cdot \cdot a}_{n-m \text{ factors}} = a^{n-m}.$$

Thus,

$$\frac{a^n}{a^m} = a^{n-m}, \text{ if } n > m.$$

Example 3

Write with a single exponent:

(a) $\frac{q^7}{q^5}$

(b) $\frac{6^7}{6^3}$

(c) $\frac{3^n}{3^4}$, where $n > 4$

(d) $\frac{\pi^5}{\pi^3}$

(e) $\frac{(c+d)^8}{(c+d)^2}$.

Solution

Since $\frac{a^n}{a^m} = a^{n-m}$ we have

(a) $\frac{q^7}{q^5} = q^{7-5} = q^2$

(b) $\frac{6^7}{6^3} = 6^{7-3} = 6^4$

(c) $\frac{3^n}{3^4} = 3^{n-4}$

(d) $\frac{\pi^5}{\pi^3} = \pi^{5-3} = \pi^2$

(e) $\frac{(c+d)^8}{(c+d)^2} = (c+d)^{8-2} = (c+d)^6$.

Just as with the products, we can write $\frac{a^n}{a^m} = a^{n-m}$ in reverse as $a^{n-m} = \frac{a^n}{a^m}$.

Example 4

Write as a quotient:

(a) 10^{2-k}

(b) e^{b-4}

(c) z^{w-s}

(d) $(p+q)^{a-b}$.

Solution

Since $a^{n-m} = \frac{a^n}{a^m}$ we have

(a) $10^{2-k} = \frac{10^2}{10^k} = \frac{100}{10^k}$

(b) $e^{b-4} = \frac{e^b}{e^4}$

(c) $z^{w-s} = \frac{z^w}{z^s}$

(d) $(p+q)^{a-b} = \frac{(p+q)^a}{(p+q)^b}$.

Raising a Power to a Power

When we take a number written in exponential form and raise it to a power, we multiply the exponents. For example,

$$(5^2)^3 = 5^2 \cdot 5^2 \cdot 5^2 = 5^{2+2+2} = 5^{2 \cdot 3} = 5^6.$$

More generally,

$$(a^m)^n = (\underbrace{a \cdot a \cdot a \cdots a}_{m \text{ factors of } a})^n = \overbrace{(\underbrace{a \cdot a \cdot a \cdots a}_{m \text{ factors of } a})(\underbrace{a \cdot a \cdot a \cdots a}_{m \text{ factors of } a}) \cdots (\underbrace{a \cdot a \cdot a \cdots a}_{m \text{ factors of } a})}^{\text{The } m \text{ factors of } a \text{ are multiplied } n \text{ times, giving a total of } m \cdot n \text{ factors of } a} = a^{m \cdot n}.$$

Thus,

$$(a^m)^n = a^{m \cdot n}.$$

Example 5

Write with a single exponent:

(a) $(q^7)^5$

(b) $(7^p)^3$

(c) $(y^a)^b$

(d) $(2^x)^x$

(e) $((x+y)^2)^3$

(f) $((r-s)^t)^z$.

Solution

Using the rule $(a^m)^n = a^{m \cdot n}$ we have

(a) $(q^7)^5 = q^{7 \cdot 5} = q^{35}$

(b) $(7^p)^3 = 7^{3p}$

(c) $(y^a)^b = y^{ab}$

(d) $(2^x)^x = 2^{x^2}$

(e) $((x+y)^2)^3 = (x+y)^{2 \cdot 3} = (x+y)^6$

(f) $((r-s)^t)^z = (r-s)^{tz}$.

Example 6

Write as a power raised to a power:

(a) $2^{3 \cdot 2}$

(b) 4^{3x}

(c) e^{4t}

(d) 6^{z^2}.

Solution

Using the rule $a^{m \cdot n} = (a^m)^n$ we have

(a) $2^{3 \cdot 2} = (2^3)^2$. This could also have been written as $(2^2)^3$.

(b) $4^{3x} = (4^3)^x$, which simplifies to 64^x. This could also have been written as $(4^x)^3$.

(c) $e^{4t} = (e^4)^t$. This could also have been written as $(e^t)^4$.

(d) $6^{z^2} = (6^z)^z$.

Products and Quotients Raised to the Same Exponent

When we multiply $5^2 \cdot 4^2$ we can change the order of the factors and rewrite it as $5^2 \cdot 4^2 = (5 \cdot 5) \cdot (4 \cdot 4) = 5 \cdot 5 \cdot 4 \cdot 4 = (5 \cdot 4) \cdot (5 \cdot 4) = (5 \cdot 4)^2 = 20^2$. Sometimes, we want to use this process in reverse: $10^2 = (2 \cdot 5)^2 = 2^2 \cdot 5^2$.

In general,

$$(a \cdot b)^n = \underbrace{(a \cdot b)(a \cdot b)(a \cdot b) \cdots (a \cdot b)}_{n \text{ factors of } (a \cdot b)} = \underbrace{\overbrace{(a \cdot a \cdot a \cdots a)}^{n \text{ factors of } a} \cdot \overbrace{(b \cdot b \cdot b \cdots b)}^{n \text{ factors of } b}}_{\text{since we can rearrange the order using the commutative property of multiplication}} = a^n \cdot b^n.$$

Thus,

$$(ab)^n = a^n b^n.$$

Example 7

Write without parentheses:

(a) $(qp)^7$

(b) $(3x)^n$

(c) $(4ab^2)^3$

(d) $(2x^{2n})^{3n}$.

Solution

Using the rule $(ab)^n = a^n b^n$ we have

(a) $(qp)^7 = q^7 p^7$

(b) $(3x)^n = 3^n x^n$

(c) $(4ab^2)^3 = 4^3 a^3 (b^2)^3 = 64a^3 b^6$

(d) $(2x^{2n})^{3n} = (2^{3n})(x^{2n})^{3n} = (2^3)^n (x)^{2n \cdot 3n} = 8^n x^{6n^2}$.

Example 8

Write with a single exponent:

(a) $c^4 d^4$

(b) $2^n \cdot 3^n$.

(c) $4x^2$

(d) $a^4 (b + c)^4$

(e) $(x^2 + y^2)^5 (c - d)^5$.

Solution

Using the rule $a^n b^n = (ab)^n$ we have

(a) $c^4 d^4 = (cd)^4$

(b) $2^n \cdot 3^n = (2 \cdot 3)^n = 6^n$

(c) $4x^2 = 2^2x^2 = (2x)^2$

(d) $a^4(b + c)^4 = (a(b + c))^4$

(e) $(x^2 + y^2)^5 (c - d)^5 = ((x^2 + y^2)(c - d))^5.$

Division of two powers with the same exponent works the same way as multiplication. For example,

$$\frac{6^4}{3^4} = \frac{6 \cdot 6 \cdot 6 \cdot 6}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{6}{3} \cdot \frac{6}{3} \cdot \frac{6}{3} \cdot \frac{6}{3} = \left(\frac{6}{3}\right)^4 = 2^4 = 16.$$

Or, reversing the process,

$$\left(\frac{4}{5}\right)^3 = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{4 \cdot 4 \cdot 4}{5 \cdot 5 \cdot 5} = \frac{4^3}{5^3}.$$

More generally,

$$\left(\frac{a}{b}\right)^n = \underbrace{\left(\frac{a}{b}\right) \cdot \left(\frac{a}{b}\right) \cdot \left(\frac{a}{b}\right) \cdots \left(\frac{a}{b}\right)}_{n \text{ factors of } a/b} = \frac{\overbrace{a \cdot a \cdot a \cdots a}^{n \text{ factors of } a}}{\underbrace{b \cdot b \cdot b \cdots b}_{n \text{ factors of } b}} = \frac{a^n}{b^n}.$$

Thus,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

Example 9

Write without parentheses:

(a) $\left(\frac{4}{5}\right)^2$

(b) $\left(\frac{c}{d}\right)^{12}$

(c) $\left(\frac{y}{z^3}\right)^4$

(d) $\left(\frac{2u}{3v}\right)^3.$

Solution

Using the rule $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ we have

(a) $\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{16}{25}$

(b) $\left(\frac{c}{d}\right)^{12} = \frac{c^{12}}{d^{12}}$

(c) $\left(\frac{y}{z^3}\right)^4 = \frac{(y)^4}{(z^3)^4} = \frac{y^4}{z^{12}}$

(d) $\left(\frac{2u}{3v}\right)^3 = \frac{(2u)^3}{(3v)^3} = \frac{2^3u^3}{3^3v^3} = \frac{8u^3}{27v^3}$.

Example 10

Write with a single exponent:

(a) $\frac{3^5}{7^5}$

(b) $\frac{q^7}{p^7}$

(c) $\frac{9x^2}{y^2}$

(d) $\frac{(p+q)^4}{z^4}$

(e) $\frac{(x^2+y^2)^5}{(a+b)^5}$

Solution

Using the rule $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ we have

(a) $\frac{3^5}{7^5} = \left(\frac{3}{7}\right)^5$

(b) $\frac{q^7}{p^7} = \left(\frac{q}{p}\right)^7$

(c) $\frac{9x^2}{y^2} = \left(\frac{3x}{y}\right)^2$

(d) $\frac{(p+q)^4}{z^4} = \left(\frac{p+q}{z}\right)^4$

(e) $\frac{(x^2+y^2)^5}{(a+b)^5} = \left(\frac{x^2+y^2}{a+b}\right)^5$.

Zero and Negative Integer Exponents

We have seen that 4^5 means 4 multiplied by itself 5 times, but what is meant by 4^0, 4^{-1} or 4^{-2}? We choose definitions for exponents like 0, -1, -2 that are consistent with the exponent rules.

If $a \neq 0$, the exponent rule for division says

$$\frac{a^2}{a^2} = a^{2-2} = a^0.$$

But $\frac{a^2}{a^2} = 1$, so we define $a^0 = 1$ if $a \neq 0$. The same idea tells us how to define negative powers. If $a \neq 0$, the exponent rule for division says

$$\frac{a^0}{a^1} = a^{0-1} = a^{-1}.$$

But $\frac{a^0}{a^1} = \frac{1}{a}$, so we define $a^{-1} = 1/a$. In general, we define

$$a^{-n} = \frac{1}{a^n}, \quad \text{if } a \neq 0.$$

Note that a negative exponent tells us to take the reciprocal of the base and change the sign of the exponent, *not* to make the number negative.

Example 11

Evaluate:

(a) 5^0

(b) 3^{-2}

(c) 2^{-1}

(d) $(-2)^{-3}$

(e) $\left(\frac{2}{3}\right)^{-1}$

Solution

(a) Any nonzero number to the zero power is one, so $5^0 = 1$.

(b) We have

$$3^{-2} = \frac{1}{3^2} = \frac{1}{9}.$$

(c) We have

$$2^{-1} = \frac{1}{2^1} = \frac{1}{2}.$$

(d) We have

$$(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{(-2)\cdot(-2)\cdot(-2)} = \frac{1}{-8} = -\frac{1}{8}.$$

(e) We have

$$\left(\frac{2}{3}\right)^{-1} = \frac{1}{\left(\frac{2}{3}\right)} = \frac{3}{2}.$$

With these definitions, we have the exponent rule for division, $\frac{a^n}{a^m} = a^{n-m}$ where n and m are integers.

Rewrite with only positive exponents. Assume all variables are positive.

Example 12

(a) $\frac{1}{3x^{-2}}$

(b) $\left(\frac{x}{y}\right)^{-3}$

(c) $\frac{3r^{-2}}{(2r)^{-4}}$

(d) $\frac{(a+b)^{-2}}{(a+b)^{-5}}$

Solution

(a) We have

$$\frac{1}{3x^{-2}} = \frac{1}{3 \cdot \frac{1}{x^2}} = \frac{1}{\frac{3}{x^2}} = \frac{x^2}{3}.$$

(b) We have

$$\left(\frac{x}{y}\right)^{-3} = \frac{1}{\left(\frac{x}{y}\right)^3} = \frac{1}{\frac{x^3}{y^3}} = \frac{y^3}{x^3}.$$

(c) We have

$$\frac{3r^{-2}}{(2r)^{-4}} = \frac{3 \cdot \frac{1}{r^2}}{\frac{1}{(2r)^4}} = \frac{\frac{3}{r^2}}{\frac{1}{16r^4}} = \frac{3}{r^2} \cdot \frac{16r^4}{1} = \frac{48r^4}{r^2} = 48r^2.$$

(d) We have

$$\frac{(a+b)^{-2}}{(a+b)^{-5}} = (a+b)^{-2+5} = (a+b)^3.$$

In part (a) of Example 12, we saw that the x^{-2} in the denominator ended up as x^2 in the numerator. In general:

$$\frac{1}{a^{-n}} = a^n \quad \text{and} \quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n.$$

Example 13 Write each of the following expressions with only positive exponents. Assume all variables are positive.

(a) $\frac{1}{4^{-2}}$

(b) $\frac{3}{4m^{-4}}$

(c) $\left(\frac{4}{5}\right)^{-2}$

(d) $\left(\frac{2x}{3y}\right)^{-3}$

(e) $\left(\frac{a+b}{a-b}\right)^{-1}$

Solution

(a) $\frac{1}{4^{-2}} = 4^2 = 16.$

(b) $\frac{3}{4m^{-4}} = \frac{3m^4}{4}.$

(c) $\left(\frac{4}{5}\right)^{-2} = \left(\frac{5}{4}\right)^2 = \frac{25}{16}.$

(d) $\left(\frac{2x}{3y}\right)^{-3} = \left(\frac{3y}{2x}\right)^3 = \frac{27y^3}{8x^3}.$

(e) $\left(\frac{a+b}{a-b}\right)^{-1} = \frac{a-b}{a+b}.$

Summary of Exponent Rules

We summarize the results of this section as follows.

general

Expressions with a Common Base

If m and n are integers,

1. $a^n \cdot a^m = a^{n+m}$
2. $\dfrac{a^n}{a^m} = a^{n-m}$
3. $(a^m)^n = a^{m \cdot n}$

Expressions with a Common Exponent

If n is an integer,

1. $(ab)^n = a^n b^n$
2. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

Zero and Negative Exponents

If a is any nonzero number and n is an integer, then:

- $a^0 = 1$
- $a^{-n} = \dfrac{1}{a^n}$

Common Mistakes

Be aware of the following notations that are sometimes confused:

$$\begin{aligned} ab^n &= a\,(b^n), && \text{but, in general,} \quad ab^n \neq (ab)^n, \\ -b^n &= -\,(b^n), && \text{but, in general,} \quad -b^n \neq (-b)^n, \\ -ab^n &= (-a)\,(b^n). \end{aligned}$$

For example, $-2^4 = -(2^4) = -16$, but $(-2)^4 = (-2)(-2)(-2)(-2) = 16$.

Example 14

Evaluate the following expressions for $x = -2$ and $y = 3$:

(a) $(xy)^4$

(b) $-xy^2$

(c) $(x + y)^2$

(d) x^y

(e) $-4x^3$

(f) $-y^2$.

Solution

(a) $(-2 \cdot 3)^4 = (-6)^4 = (-6)(-6)(-6)(-6) = 1296$.

(b) $-(-2) \cdot (3)^2 = 2 \cdot 9 = 18$.

(c) $(-2 + 3)^2 = (1)^2 = 1$.

(d) $(-2)^3 = (-2)(-2)(-2) = -8$.

(e) $-4(-2)^3 = -4(-2)(-2)(-2) = 32$.

(f) $-(3)^2 = -9$.

Problems for Section 6.1

EXERCISES

Evaluate the expressions in Exercises 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 without using a calculator.

1. $3 \cdot 2^3$

Answer:

24

2. -3^2

3. $(-2)^3$

Answer:

-8

4. $5^1 \cdot 1^4 \cdot 3^2$

5. $5^2 \cdot 2^2$

Answer:

100

6. $\dfrac{-1^3 \cdot (-3)^4}{9^2}$

7. $(-5)^3 \cdot (-2)^2$

Answer:

-500

8. $-5^3 \cdot -2^2$

9. $-1^4 \cdot (-3)^2(-2^3)$

Answer:

72

10. $\left(\frac{-4^3}{-2^3}\right)^2$

11. 3^0

Answer:

1

12. 0^3

In Exercises 13, 14, 15, 16, 17, 18, 19, 20, 21 and 22, evaluate the following expressions for $x = 2$, $y = -3$, and $z = -5$.

13. $-xyz$

Answer:

-30

14. y^x

15. $-y^x$

Answer:

-9

16. $\left(\frac{y}{z}\right)^x$

17. $\left(\frac{x}{z}\right)^{-y}$

Answer:

-8/125

18. x^{-z}

19. $-x^{-z}$

Answer:

-32

20. $\left(\frac{x^3 y}{2z}\right)^2$

21. $\left(\frac{3y}{2z}\right)^3$

Answer:

729/1000

22. $\left(\frac{x+y}{x-z}\right)^x$

In Exercises 23, 24, 25, 26, 27, 28, 29, 30 and 31 , write the expression in the form x^n, assuming $x \neq 0$.

23. $x^3 \cdot x^5$

Answer:

x^8

24. $\frac{x^3 \cdot x^4}{x^2}$

25. $(x^4 \cdot x)^2$

Answer:

x^{10}

26. $\left(\frac{x^5}{x^2}\right)^4$

27. $\frac{x^5 \cdot x^3}{x^4 \cdot x^2}$

Answer:

x^2

28. $\frac{x^7}{x^4} \cdot \frac{x^5}{x}$

29. $(x^3)^5$

Answer:

x^{15}

30. $\frac{x \cdot x^6}{(x^3)^2}$

31. $\frac{(x^4 \cdot x^6)^2}{(x^2 \cdot x^3)^3}$

Answer:

x^5

In Exercises 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44 and 45, write with a single exponent.

32. $4^2 \cdot 4^n$

33. $2^n 2^2$

Answer:

2^{n+2}

34. $a^5 b^5$

35. $\frac{a^x}{b^x}$

Answer:

$(a/b)^x$

36. $\frac{2^a 3^a}{6^b}$

37. $\frac{4^n}{2^m}$

Answer:

$2^{2n - m}$

38. $A^{n+3} B^n B^3$

39. $B^a B^{a+1}$

Answer:

B^{2a+1}

40. $(x^2 + y)^3 (x + y^2)^3$

41. $((x+y)^4)^5$

Answer:

$(x + y)^{20}$

42. $16^2 y^8$

43. $\frac{(g+h)^6 (g+h)^5}{((g+h)^2)^4}$

Answer:

$(g + h)^3$

44. $((a+b)^2)^5$

45. $\dfrac{(a+b)^5}{(a+b)^2}$

Answer:

$(a+b)^3$

Without a calculator, decide whether the quantities in Exercises 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58 and 59 are positive or negative

46. $(-4)^3$

47. -4^3

Answer:

Negative

48. $(-3)^4$

49. -3^4

Answer:

Negative

50. $(-23)^{42}$

51. -31^{66}

Answer:

Negative

52. 17^{-1}

53. $(-5)^{-2}$

Answer:

Positive

54. -5^{-2}

55. $(-4)^{-3}$

Answer:

Negative

56. $(-73)^0$

57. -48^0

Answer:

Negative

58. $(-47)^{-15}$

59. $(-61)^{-42}$

Answer:

Positive

In Exercises 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71 and 72, write each expression without parentheses. Assume all variables are positive.

60. $\left(\frac{a}{b}\right)^5$

61. $\left(\frac{c^3}{d}\right)^4$

Answer:

c^{12}/d^4

62. $\left(\frac{2p}{q^3}\right)^5$

63. $\left(\frac{4r^2}{5s^4}\right)^3$

Answer:

$64r^6/125s^{12}$

64. $\left(\frac{3}{w^4}\right)^4$

65. $\left(\frac{6g^5}{7h^7}\right)^2$

Answer:

$36g^{10}/49h^{14}$

66. $(cf)^9$

67. $(2p)^5$

Answer:

$32p^5$

68. $\left(\frac{2}{3}\right)^4$

69. $\left(4b^2\right)^{2t}$

Answer:

$16^t b^{4t}$

70. $3\left(10e^{3t}\right)^2$

71. $3\left(2^x e^x\right)^4$

Answer:

$3 \cdot 16^x e^{4x}$

72. $\left(3x^2\right)^{2n}$

PROBLEMS

In Problems 73, 74, 75, 76 and 77, decide which expressions are equivalent. Assume all variables are positive.

73.

(a) 3^{-2}

(b) $\frac{1}{3^{-2}}$

(c) $\frac{1}{3^2}$

(d) $\left(\frac{1}{3}\right)^2$

(e) $\left(\frac{1}{3}\right)^{-2}$

Answer:

(a), (c), (d) equivalent; (b), (e) equivalent

74.

(a) $\left(\frac{2}{3}\right)^{-n}$

(b) $\left(\frac{1}{\frac{2}{3}}\right)^n$

(c) $\left(\frac{3}{2}\right)^n$

(d) $-\left(\frac{2}{3}\right)^n$

(e) $\frac{2^{-n}}{3^{-n}}$

75.

(a) $\frac{1}{x^{-r}}$

(b) $\frac{1}{x^r}$

(c) $\left(\frac{1}{x}\right)^{-r}$

(d) x^{-r}

(e) $\frac{1}{\frac{1}{x^r}}$

Answer:

(a), (c), (e) equivalent; (b), (d) equivalent

76.

(a) $\frac{1}{\left(\frac{r}{s}\right)^{-t}}$

(b) $\left(\frac{s}{r}\right)^{-t}$

(c) $\frac{1}{\left(\frac{r}{s}\right)^{t}}$

(d) $\left(r^{-t}\right)\frac{1}{s^{-t}}$

(e) $\left(rs^{-1}\right)^{t}$

77.

(a) $\left(\frac{p}{q}\right)^{-m}$

(b) $\left(\frac{\frac{1}{p}}{\frac{1}{q}}\right)^{m}$

(c) $\left(\frac{p^{-1}}{q^{-1}}\right)^{-m}$

(d) $\left(p^{-1}q\right)^{-m}$

(e) $\left(\frac{\frac{1}{q^{-1}}}{\frac{1}{p^{-1}}}\right)^{-m}$

Answer:

(a), (b) equivalent; (c), (d), (e) equivalent

In Problems 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90 and 91 , write each expression as a product or a quotient. Assume all variables are positive.

78. 3^{2+3}

79. a^{4+1}

Answer:

$a^{4+1} = a^4 \cdot a$

80. e^{2+r}

81. 10^{4-z}

Answer:

$10^4/10^z$

82. k^{a-b}

83. 4^{p+3}

Answer:

$4^p \cdot 4^3$

84. 6^{a-1}

85. $(-n)^{a+b}$

Answer:

$(-n)^a(-n)^b$

86. x^{a+b+1}

87. $p^{1-(a+b)}$

Answer:

$p/(p^a p^b)$

88. $(r-s)^{t+z}$

89. $(p+q)^{a-b}$

Answer:

$(p+q)^a/(p+q)^b$

90. $e^{t-1}(t+1)$

91. $(x+1)^{ab+c}$

Answer:

$(x+1)^{ab}(x+1)^c$

In Problems 92, 93, 94, 95, 95, 96, 97 and 98, write each expression as a power raised to a power. There may be more than one correct answer.

92. $4^{2 \cdot 4}$

93. 2^{3x}

Answer:

$(2^3)^x = 8^x$

94. 5^{2y}

95. 3^{4a}

Answer:

$\left(3^4\right)^a = 81^a$

96. 5^{x^2}

97. $3e^{2t}$

Answer:

$\left(\sqrt{3}e^t\right)^2$

98. $(x + 3)^{2w}$

99. If $3^a = w$, express 3^{3a} in terms of w.

Answer:

w^3

100. If $3^x = y$, express 3^{x+2} in terms of y.

101. If $4^b = c$, express 4^{b-3} in terms of c.

Answer:

$c/64$

102. If $x^a = \frac{y}{z}$, $y = x^b$, and $z = x^c$, what is a?