EVAL 6970: Meta-Analysis
Effect Sizes and Precision: Part I

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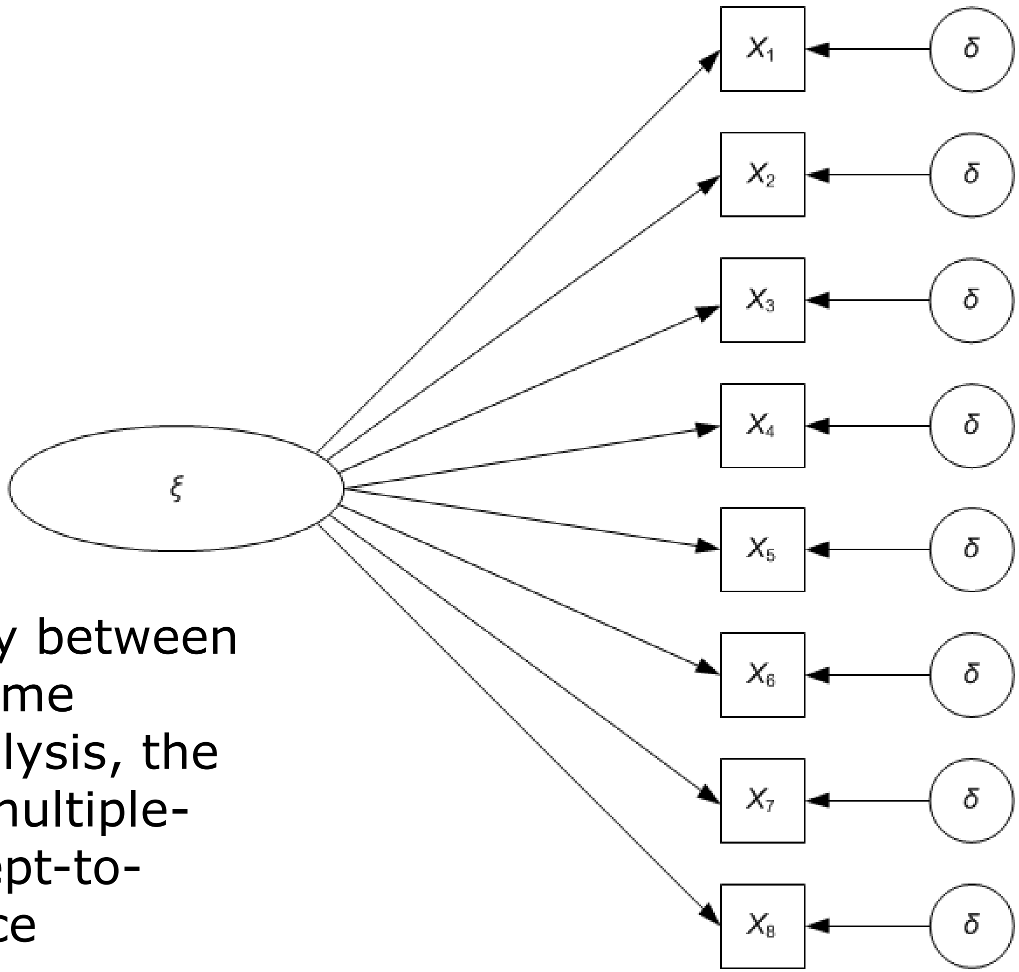
Agenda

- Effect sizes based on means
- Review questions
- In-class activity

Note

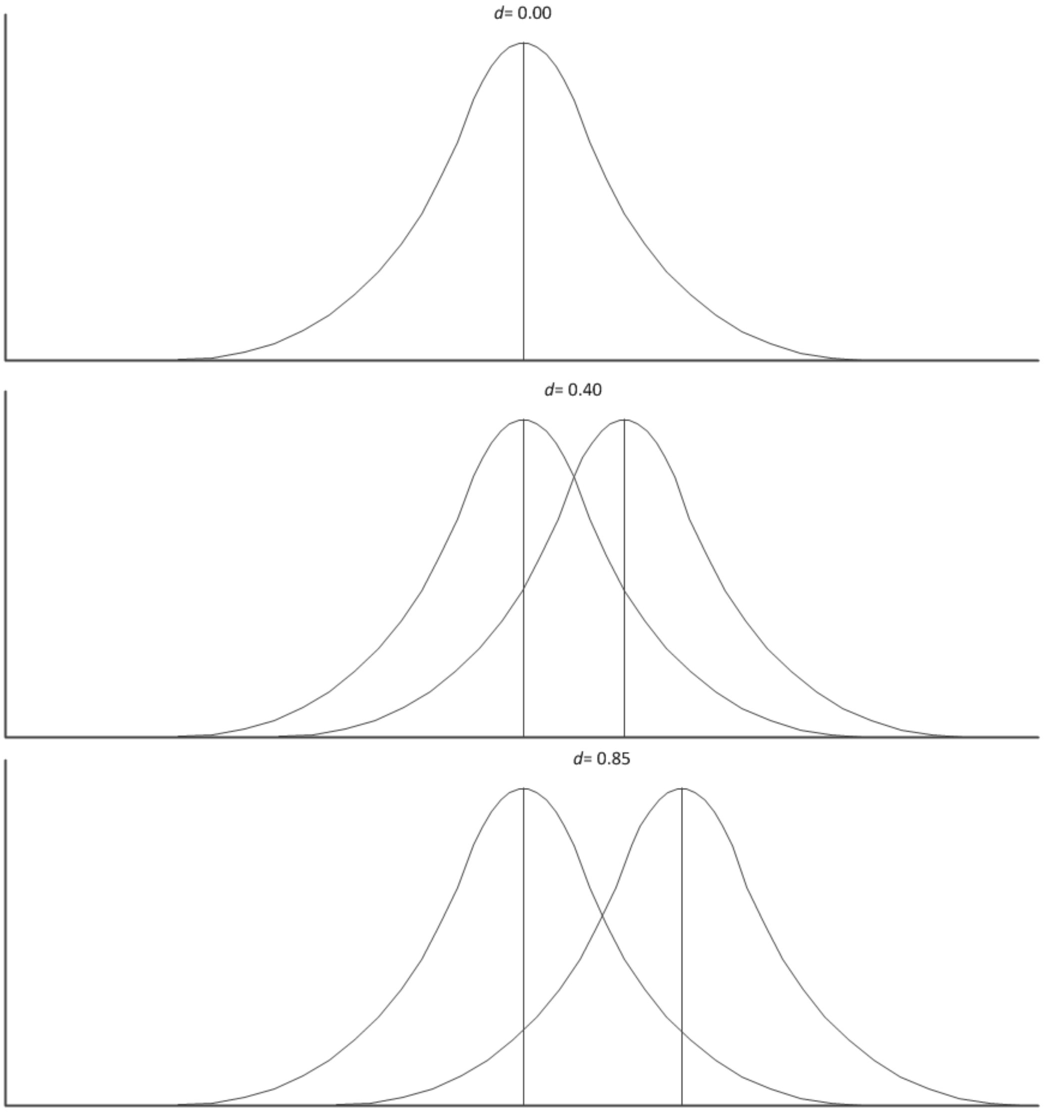
- Statistical notation varies widely for the topics covered today and in future lectures
- By convention (and for consistency), we will predominately use Borenstein, Hedges, Higgins, and Rothstein's (2010) notation

The Apples and Oranges Argument: Both are Fruits



There is a useful analogy between including different outcome measures in a meta-analysis, the common factor model, multiple-operationism, and concept-to-operation correspondence

Effect Sizes



Raw Mean Difference, *D*

- The raw (unstandardized) mean difference can be used on a meaningful outcome measure (e.g. blood pressure) and when all studies use the same measure
- The population mean difference is defined as

$$\Delta = \mu_1 - \mu_2$$

D from Independent Groups

- The mean difference Δ from studies using two independent groups (e.g., treatment and control) can be estimated from the sample group means $\bar{X}_1$ and $\bar{X}_2$ as

$$D = \bar{X}_1 - \bar{X}_2$$

D from Independent Groups

- Assuming $\sigma_1 = \sigma_2 = \sigma$ (as is assumed for most parametric statistics) the variance of *D* is

$$V_D = \frac{n_1 + n_2}{n_1 n_2} S_{pooled}^2$$

- where

$$S_{pooled} = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

D from Independent Groups

- Assuming $\sigma_1 \neq \sigma_2$ the variance of *D* is

$$V_D = \frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}$$

- For both $\sigma_1 = \sigma_2$ and $\sigma_1 \neq \sigma_2$ the standard error of *D* is

$$SE_D = \sqrt{V_D}$$

D from Dependent Groups

- When groups are dependent (e.g., matched pairs designs or pretest-posttest designs) then *D* is the difference score for each pair

$$D = \bar{X}_{diff} \text{ or } D = \bar{X}_1 - \bar{X}_2$$

D from Dependent Groups

- Where the variance is

$$V_D = \frac{S^2_{diff}}{n}$$

- Where *n* is the number of pairs, and

$$SE_D = \sqrt{V_D}$$

D from Dependent Groups

- If S_{diff} must be computed

$$S_{diff} = \sqrt{S_1^2 + S_2^2 - 2 \times r \times S_1 \times S_2}$$

- Where *r* is the correlation between pairs
- If $S_1 = S_2$ then

$$S_{diff} = \sqrt{2 \times S_{pooled}^2 (1 - r)}$$

Standardized Mean Difference, *d* and *g*

- Assuming $\sigma_1 = \sigma_2 = \sigma$ (as is the case for most parametric statistics) the standardized mean difference population parameter is defined as

$$\delta = \frac{\mu_1 - \mu_2}{\sigma}$$

d and *g* from Independent Groups

- The standardized mean difference (δ) from independent groups (e.g., treatment and control) can be estimated from the sample group means $\bar{X}_1$ and $\bar{X}_2$ as

$$d = \frac{\bar{X}_1 - \bar{X}_2}{S_{within}}$$

d and *g* from Independent Groups

- Where S_{within} is the within-groups standard deviation, pooled across groups

$$S_{within} = \sqrt{\frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}}$$

d and *g* from Independent Groups

- Where the variance is

$$V_d = \frac{n_1 + n_2}{n_1 n_2} + \frac{d^2}{2(n_1 + n_2)}$$

- And the standard error is

$$SE_d = \sqrt{V_d}$$

d and *g* from Independent Groups

- In small samples (N < 20), d overestimates δ and this bias can be reduced by converting d to Hedges' g using the correction factor J, where

$$J = 1 - \frac{3}{4df - 1}$$

- For two independent groups

$$df = n_1 + n_2 - 2$$

d and *g* from Independent Groups

- Using the correction factor *J*, Hedges' *g* is calculated as

$$g = J \times d$$

- With

$$V_g = J^2 \times V_d$$

- And

$$SE_g = \sqrt{V_g}$$

d and *g* from Dependent Groups

- When groups are dependent (e.g., matched pairs designs or pretest-posttest designs) then *d* is the difference score for each pair

$$d = \frac{\bar{Y}_{diff}}{S_{within}} = \frac{\bar{Y}_1 - \bar{Y}_2}{S_{within}}$$

d and *g* from Dependent Groups

- Where S_{within} is

$$S_{within} = \frac{S_{diff}}{\sqrt{2(1-r)}}$$

d and *g* from Dependent Groups

- Where the variance is

$$V_d = \left(\frac{1}{n} + \frac{d^2}{2n}\right) 2(1 - r)$$

- Where *n* is the number of pairs, and

$$SE_D = \sqrt{V_d}$$

d and *g* from Dependent Groups

- If S_{diff} must be computed

$$S_{diff} = \sqrt{S_1^2 + S_2^2 - 2 \times r \times S_1 \times S_2}$$

- Where *r* is the correlation between pairs
- If $S_1 = S_2$ then

$$S_{diff} = \sqrt{2 \times S_{pooled}^2 (1 - r)}$$

d and *g* from Dependent Groups

- In small samples (N < 20), *d* overestimates δ and this bias can be reduced by converting *d* to Hedges' *g* using the correction factor J, where

$$J = 1 - \frac{3}{4df - 1}$$

- Where, in dependent groups

$$df = n - 1$$

d and *g* from Dependent Groups

- Using the correction factor *J*, Hedges' *g* is calculated as

$$g = J \times d$$

- With

$$V_g = J^2 \times V_d$$

- And

$$SE_g = \sqrt{V_g}$$

Effect Direction

- For all designs the direction of the effect ($\bar{X}_1 - \bar{X}_2$ or $\bar{X}_2 - \bar{X}_1$) is arbitrary, except that the same convention must be applied to all studies in a meta-analysis (e.g., a positive difference indicates that the treated group did better than the control group)

Review Questions

1. When is it appropriate to use D?

2. When is it appropriate to use d?

3. When is it appropriate to use g?

Today's In-Class Activity

- Individually, or in your working groups, download "Data Sets 1-6 XLSX" from the course Website
  - Calculate the appropriate effects sizes, standard deviations, variances, and standard errors for Data Sets 1, 2, 3, and 4
  - Be certain to save your work as we will use these data again