EVAL 6970: Meta-Analysis
Effect Sizes and Precision: Part II

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Fall 2013

Agenda

- Effect sizes based on binary data
- Effect sizes based on correlations
- Converting among effect sizes
- Precision
- Review questions
- In-class activity

2×2 Tables for Binary Data

- For risk ratios, odds ratios, and risk differences data are typically represented in 2×2 tables with cells A, B, C, and D

	Events	Non-Events	N
Treated	A	B	n_1
Control	C	D	n_2

Risk Ratios

- The risk ratio is the ratio of two risks where

$$\text{Risk Ratio} = \frac{A/n_1}{C/n_2}$$

Risk Ratios

- For the purpose of meta-analysis, computations are conducted using a log scale where

$$\text{Log Risk Ratio} = \ln(\text{Risk Ratio})$$

Risk Ratios

- With variance

$$V_{\text{Log Risk Ratio}} = \frac{1}{A} - \frac{1}{n_1} + \frac{1}{C} - \frac{1}{n_2}$$

- And standard error

$$SE_{\text{Log Risk Ratio}} = \sqrt{V_{\text{Log Risk Ratio}}}$$

Odds Ratios

- The odds ratio is the ratio of two odds where

$$\text{Odds Ratio} = \frac{AD}{BC}$$

Odds Ratios

- For the purpose of meta-analysis, computations are conducted using a log scale where

$$\text{Log Odds Ratio} = \ln(\text{Odds Ratio})$$

Odds Ratios

- With variance

$$V_{\text{Log Odds Ratio}} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C} + \frac{1}{D}$$

- And standard error

$$SE_{\text{Log Odds Ratio}} = \sqrt{V_{\text{Log Odds Ratio}}}$$

Risk Difference

- The risk difference is the difference between two risks where

$$\text{Risk Difference} = \left(\frac{A}{n_1}\right) - \left(\frac{C}{n_2}\right)$$

- For risk differences all computations are performed on the raw units

Risk Difference

- With variance

$$V_{\text{Risk Difference}} = \frac{AB}{n_1^3} + \frac{CD}{n_2^3}$$

- And standard error

$$SE_{\text{Risk Difference}} = \sqrt{V_{\text{Risk Difference}}}$$

Correlation Coefficient *r*

- The estimate of the correlation population parameter ρ is r where

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}}$$

- With variance

$$V_r = \frac{(1 - r^2)^2}{n - 1}$$

Correlation Coefficient *r*

- For meta-analyses, *r* is converted to Fisher's *z*
- The transformation of *r* to *z* is

$$z = 0.5 \times \ln\left(\frac{1+r}{1-r}\right)$$

Correlation Coefficient *r*

- With variance

$$V_z = \frac{1}{n-3}$$

- And standard error

$$SE_z = \sqrt{V_z}$$

Converting Among Effect Sizes

- Often, different studies report different effect sizes (if at all) and for a meta-analysis all effect sizes need to be converted to a common index
- Meta-Analysis 2.0 will automate this process and many effect sizes calculators are also useful

Converting from Odds Ratio to *d*

- To convert from the log odds ratio to *d*

$$d = \text{Log Odds Ratio} \times \frac{\sqrt{3}}{\pi}$$

- With variance of

$$V_d = V_{\text{Log Odds Ratio}} \times \frac{3}{\pi^2}$$

Converting from *d* to Odds Ratio

- To convert from *d* to the log odds ratio

$$\text{Log Odds Ratio} = d \times \frac{\pi}{\sqrt{3}}$$

- With variance of

$$V_{\text{Log Odds Ratio}} = V_d \times \frac{\pi^2}{3}$$

Converting from *r* to *d*

- To convert from *r* to *d*

$$d = \frac{2r}{\sqrt{1 - r^2}}$$

- With variance of

$$V_d = \frac{4V_r}{(1 - r^2)^2}$$

Converting from *d* to *r*

- To convert from *d* to *r*

$$r = \frac{d}{\sqrt{d} = a}$$

- Where *a* is a correction factor when $n_1 \neq n_2$

$$a = \frac{(n_1 + n_2)^2}{n_1 n_2}$$

Converting from *d* to *r*

- With variance

$$V_r = \frac{a^2 V_d}{(d^2 + a)^3}$$

Precision

- Provides the context for computing standard errors
- Precision includes variance, standard error, and confidence intervals
- With variance V_Y the standard error $SE_Y = \sqrt{V_Y}$ can be computed
  - For different effect size metrics, the computation of V_Y differs

Confidence Intervals

- Assuming that an effect size is normally distributed

$$LL_Y = \bar{Y} - 1.96 \times SE_Y$$

- And

$$UL_Y = \bar{Y} + 1.96 \times SE_Y$$

- 1.96 is the *Z*-value corresponding to confidence limits of 95% (with error of 2.5% at either end of the distribution)

Review Questions

1. When is it appropriate to use the risk ratio?
2. When is it appropriate to use the odds ratio?
3. When is it appropriate to use the risk difference?
4. When is it appropriate to use r?
5. What factors affect precision and how?

Today's In-Class Activity

- Individually, or in your working groups, download "Data Sets 1-6 XLSX" from the course Website
  - Calculate the appropriate effects sizes, standard deviations, variances, and standard errors for Data Sets 5 and 6
  - Calculate the 95% confidence intervals (i.e., *LL* and *UL*) for Data Sets 1, 2, 3, 4, 5, and 6
  - Be certain to save your work as we will use these data again