EVAL 6970: Meta-Analysis

Fixed-Effect and Random-Effects Models

Dr. Chris L. S. Coryn

Kristin A. Hobson

Fall 2013

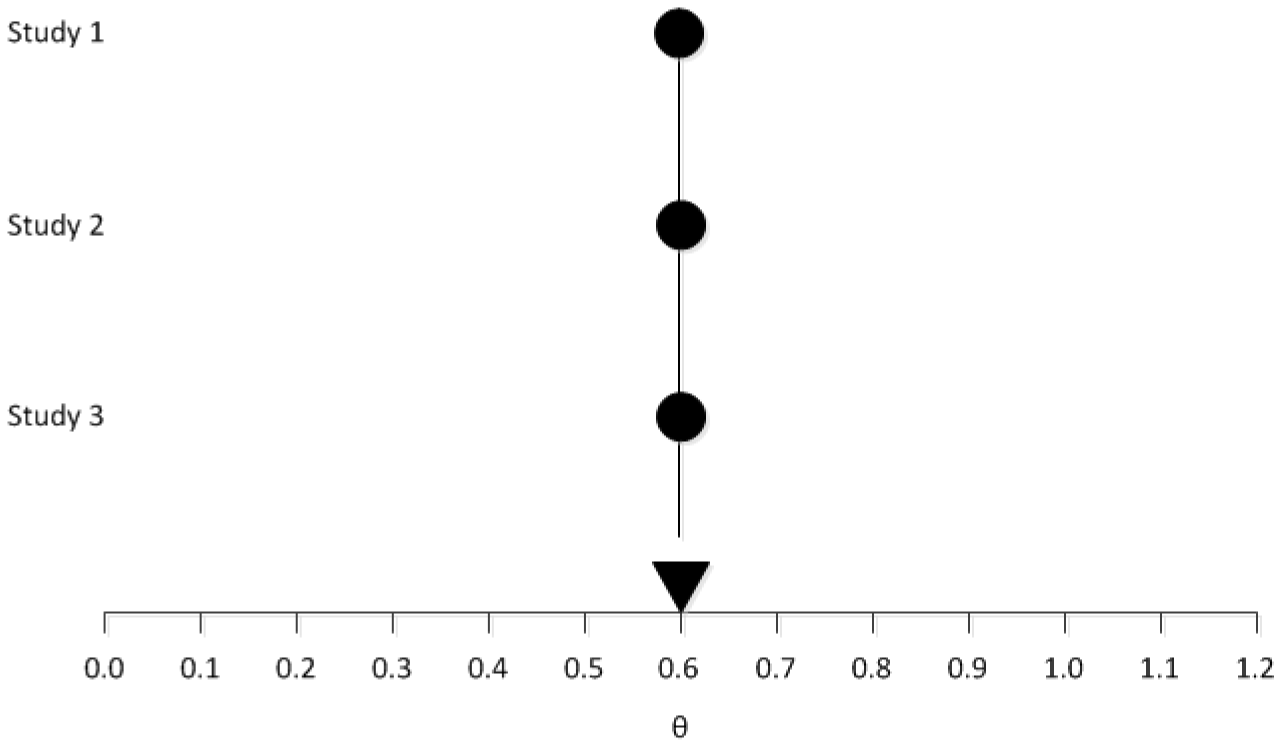
Agenda

- Fixed-effect models for meta-analysis
- Random-effects models for meta-analysis
- Review questions
- In-class activity

Fixed-Effect Models

- Under the fixed-effect model it is assumed that all studies share a common (true) effect size
- All factors that could influence the effect size are the same across all studies (the true effect is the same, thus *fixed*)
- The true (unknown) effect size is θ

Fixed-Effect Models

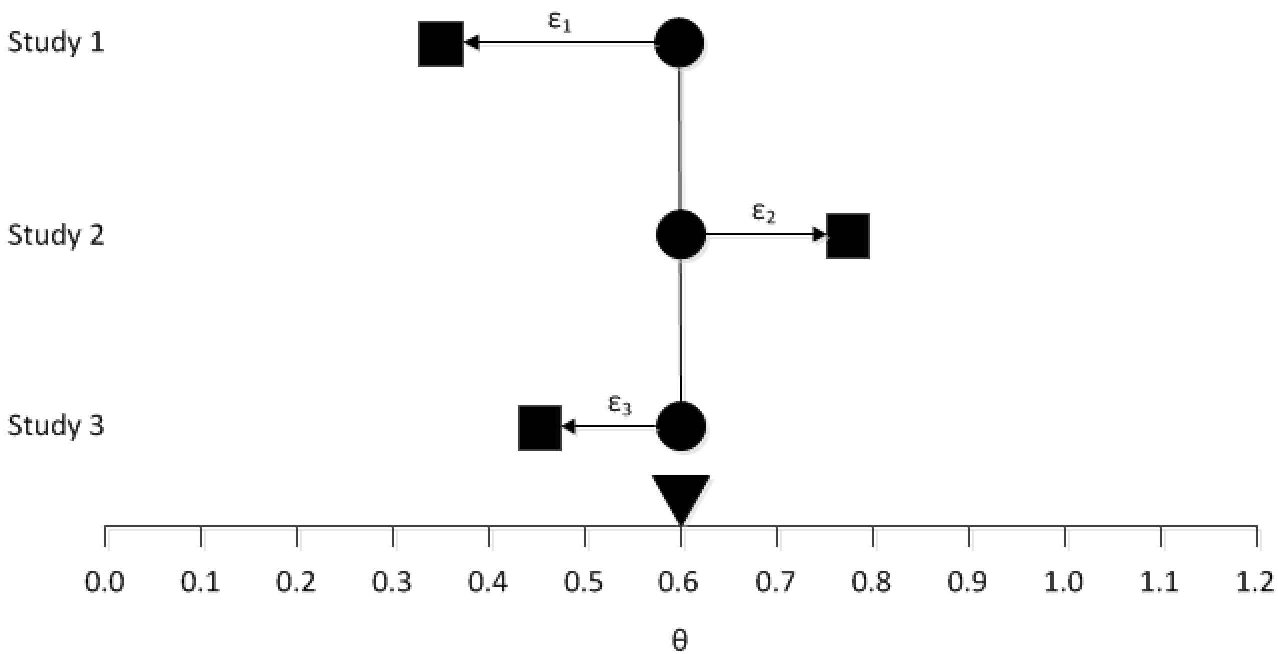


Fixed-effect model: True effect

Fixed-Effect Models

- Given that all studies share the same true effect, the observed effect size varies from study to study only because of random (sampling) error
- Although error is random, the sampling distribution of the errors can be estimated

Fixed-Effect Models



$$Y_i = \theta + \varepsilon_i$$

Fixed-effect model: True effects and sampling error

Fixed-Effect Models

- To obtain an estimate of the population effect (to minimize variance), a weighted mean is computed using the inverse of each study's variance as the study's weight in computing the mean effect

Fixed-Effect Models

- The weighted mean (M) is computed as

$$M = \frac{\sum_{i=1}^{k} W_i Y_i}{\sum_{i=1}^{k} W_i}$$

- That is, the sum of the products $W_i Y_i$ (effect size multiplied by weight) divided by the sum of the weights

Fixed-Effect Models

- Where the weight assigned to each study is

$$W_i = \frac{1}{V_{Y_i}}$$

- Where V_{Y_i} is the within-study variance for study i

Fixed-Effect Models

- With

$$V_M = \frac{1}{\sum_{i=1}^{k} W_i}$$

- And

$$SE_M = \sqrt{V_M}$$

- And

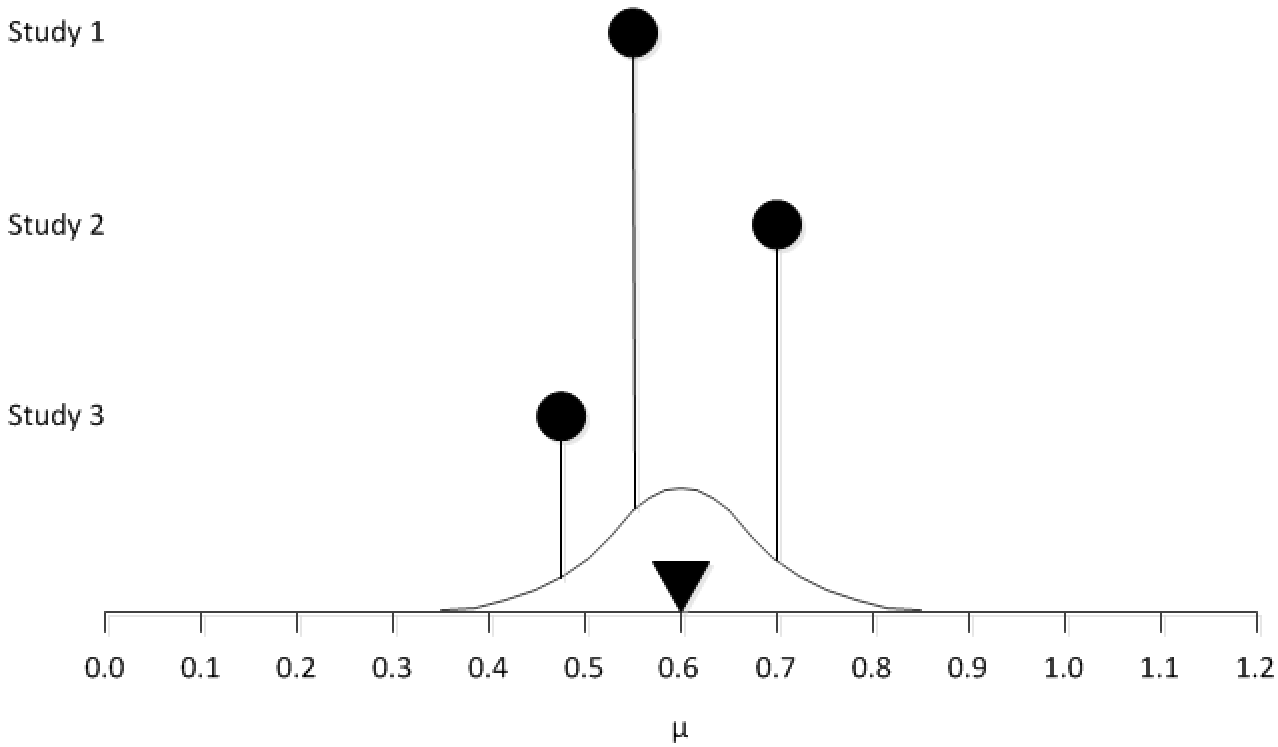
$$LL_M = M - 1.96 \times SE_M$$

$$UL_M = M + 1.96 \times SE_M$$

Random-Effects Models

- Does not assume that the true effect is identical across studies
- Because study characteristics vary (e.g., participant characteristics, treatment intensity, outcome measurement), there may be different effect sizes underlying different studies

Random-Effects Models

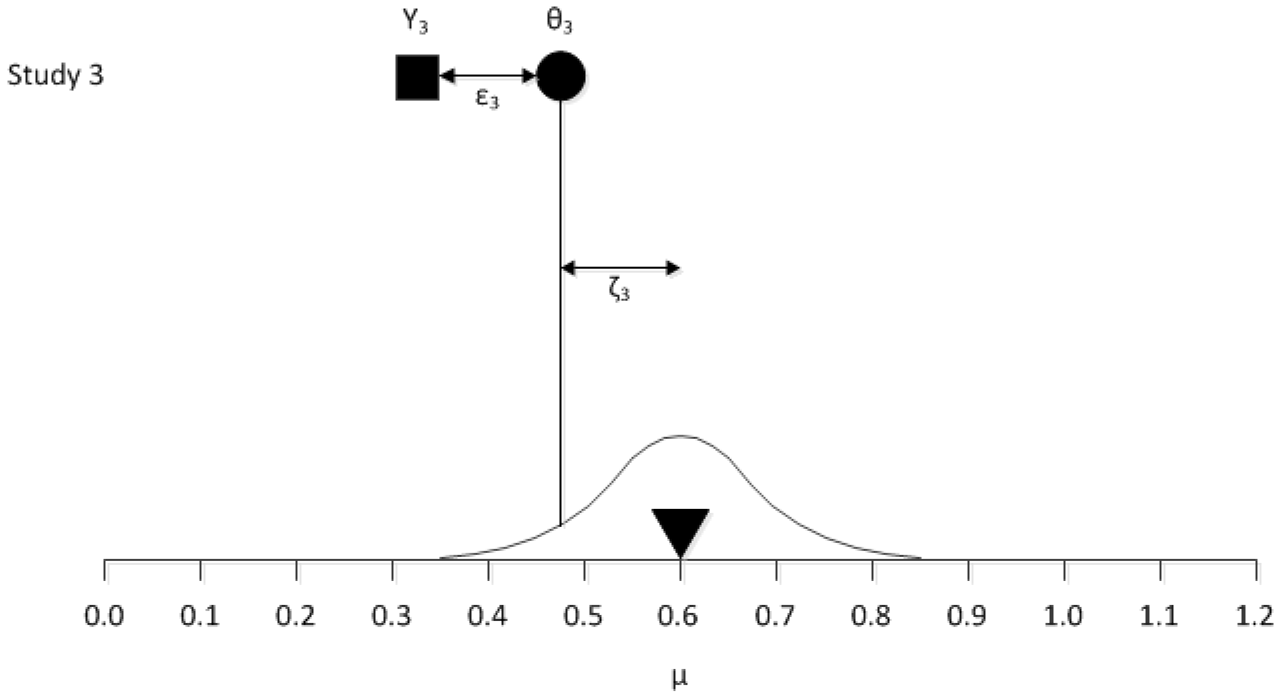


Random-effects model: True effects

Random-Effects Models

- If the true effect size for a study is θ_i, then the observed effect will be less than or greater than θ_i due to sampling error
- The distance between the summary mean and the observed effect consists of true variation in effect sizes (ζ_i) and sampling error (ε_i)

Random-Effects Models



$$Y_i = \mu + \zeta_i + \varepsilon_i$$

Random-effects model: True effect and observed effect

Random-Effects Models

- The distance from μ to each θ_i depends on the standard deviation of the true effects across studies, which is represented as τ and τ^2 for its variance
- Each study's variance is a function of both the within-study variance and τ^2 and is the sum of these two values

Random-Effects Models

- τ^2 is estimated as

$$T^2 = \frac{Q - df}{C}$$

- Where

$$Q = \sum_{i=1}^{k} W_i Y_i^2 - \frac{\left(\sum_{i=1}^{k} W_i Y_i\right)^2}{\sum_{i=1}^{k} W_i}$$

Random-Effects Models

- With

$$df = k - 1$$

- Where k is the number of studies

- And

$$C = \sum W_i - \frac{\sum W_i^2}{\sum W_i}$$

Random-Effects Model

- Under the random-effects model, the weight assigned to each study is

$$W_i^* = \frac{1}{V_{Y_i}^*}$$

- Where $V_{Y_i}^*$ is the within-study variance for study i plus the between study variance T^2

Random-Effects Models

- Where

$$V^{*}_{Y_i} = V_{Y_i} + T^2$$

- With the weighted mean M^* computed as

$$M^* = \frac{\sum_{i=1}^{k} W_i^* Y_i}{\sum_{i=1}^{k} W_i^*}$$

Random-Effects Models

- With

$$V_{M^*} = \frac{1}{\sum_{i=1}^{k} W_i^*}$$

- And

$$SE_{M^*} = \sqrt{V_{M^*}}$$

- And

$$LL_{M^*} = M^* - 1.96 \times SE_{M^*}$$

$$UL_{M^*} = M^* + 1.96 \times SE_{M^*}$$

Review Questions

1. When is it appropriate to use a fixed-effect model?
2. When is it appropriate to use a random-effects model?
3. How do the study weights differ for fixed-effect and random-effects models?

Today's In-Class Activity

- Extracting effect size information from primary studies
- Individually, using the study provided
  - Code the study design
  - Calculate the OR for 'recidivism'
  - Calculate *d* for 'recidivism'
  - Compare your estimates to those of others in the course

Today’s In-Class Activity

- From “Data Sets 1-6 XLSX”
  - Calculate the fixed-effect and random-effects model weighted means for Data Sets 3 and 5
  - Calculate the 95% confidence intervals (i.e., *LL* and *UL*) for the weighted means from Data Sets 3 and 5

Today's In-Class Activity

- Download "Meta-Analysis with Means, Binary Data, and Correlations XLSX" from the course Website
  - Verify your results from the prior exercise
- Enter Data Sets 3 and 5 (as separate files) into Comprehensive Meta-Analysis 2.0
  - Again, verify your results

Today's In-Class Activity

- Compare the results of the fixed-effect and random-effects meta-analyses
  - How similar or different are the results for the two models?
  - How might you explain these similarities or differences?