EVAL 6970: Meta-Analysis
Heterogeneity and Prediction Intervals

Dr. Chris L. S. Coryn
Kristin A. Hobson
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Agenda

- Identifying and quantifying heterogeneity
  - Review questions
  - In-class activity
- Prediction intervals
  - Review questions
  - In-class activity

Quantifying Heterogeneity

- Although we are usually concerned with the dispersion in true effect sizes, observed dispersion includes both true variance and random error
- The mechanism used to isolate true variance is to compare the observed dispersion with the amount expected if all studies shared a common effect size
  - The excess is assumed to reflect real differences among studies
  - This portion of the variance is used to create indices of heterogeneity

Indices of Heterogeneity

- The Q statistic (the weighted sum of squares; *WSS*) and its *p*-value serve as a statistical test of significance (compared to the expected *WSS*) and address the viability of the null hypothesis that the true dispersion is exactly zero, and not the excess amount of dispersion
  - Sensitive to the number of studies and is metric-free (i.e., the effect size index)

Indices of Heterogeneity

- The estimate of τ^2 (i.e., T^2) serves as the between-study variance and τ (i.e., T) serves as the standard deviation of the true effects and both describe the distribution of true effects
  - They are sensitive to the effect size metric, but not the number of studies
  - 95% LL and UL can be computed for both T^2 and T

Indices of Heterogeneity

- T is useful for understanding the range of effects and is on the same metric as the effects
  - If a summary effect were 0.50 and T was 0.10, most effects fall in the range of 0.30 to 0.70
  - If a summary effect were 0.50 and T was 0.20, then most effects fall in the range of 0.10 and 0.90
  - This index is useful regarding interpretation (e.g., utility of the intervention)

Indices of Heterogeneity

- I^2 provides information regarding what proportion of observed variance is real and is a ratio with a range from 0% to 100%
  - An I^2 near zero indicates that the dispersion is attributable to random error, and any attempt to explain the variance is an attempt to explain something that is random
  - As I^2 increases, some of the variance is real and can potentially be explained by subgroup analysis or meta-regression
  - 95% LL and UL (uncertainty intervals) can be computed for I^2

Factors Affecting Heterogeneity Statistics

	Range of possible values	Depends on number of studies	Depends on scale
Q	$0 \leq Q$	✓	
p	$0 \leq p \leq 1$	✓	
T^2	$0 \leq T^2$		✓
T	$0 \leq T$		✓
I^2	$0\% \leq I^2 < 100\%$		

- All indices are based on Q (in relation to df)
- Each is useful for a specific purpose

The Q Statistic

- The first step in partitioning study-to-study variation is to compute Q

$$Q = \sum_{i=1}^{k} W_i (Y_i - M)^2$$

- or

$$Q = \sum_{i=1}^{k} \left(\frac{Y_i - M}{S_i} \right)^2$$

The Expected Value of Q

- Next, determine the expected value of Q (on the assumption that all studies share a common effect size)
- Because Q is a standardized measure the expected value does not depend on the metric of the effect size, the expected value (WSS) is

$$df = k - 1$$

Using Q to Test Homogeneity of Effects

- To test the null hypothesis that all studies share a common effect size use Q and df from previous slides
- Q will follow a central chi-squared distribution with degrees of freedom equal to $k - 1$, and a p-value for any observed value of Q can be obtained

$$=\text{CHIDIST}(Q, df)$$

Estimating τ^2

- τ^2 is the variance of the true effect sizes
- Since τ^2 cannot be directly computed, it is estimated with T^2

$$T^2 = \frac{Q - df}{C}$$

- Where

$$C = \sum W_i - \frac{\sum W_i^2}{\sum W_i}$$

Estimating τ

- τ is the standard deviation of the true effect sizes and the estimate of this parameter is T

$$T = \sqrt{T^2}$$

- T is on the same scale as the effect size and can be used to describe the distribution of effect sizes

Estimating I^2

- I^2 represent the proportion of observed variance that reflect real differences in effect sizes and is the ratio of excess dispersion to total dispersion

$$I^2 = \left(\frac{Q - df}{Q}\right) \times 100\%$$

Estimating I^2

- I^2 reflects the overlap of the study confidence intervals, which is not dependent on the actual location or dispersion of the true effects
- Therefore, I^2 is a measure of inconsistency across the findings of the studies, and not real variation across the underlying true effects

Review Questions

1. What question does Q address?
2. What information does T^2 and T provide?
3. What information does I^2 provide and how is it interpreted?

Today’s First In-Class Activity

- Using “Data Sets for Heterogeneity Analysis XLSX” from the course Website
  - Calculate the heterogeneity statistics for Step 5 and Step 9 (except for the prediction intervals) for each data set
  - In some cases, you will need to refer to the book (formulas and page numbers are provided)
  - Verify your results using Meta-Analysis 2.0
  - Interpret the heterogeneity statistics

Prediction Intervals

- The precision of the summary effect is addressed by the LL and UL confidence intervals, which reflects only error (V_{M^*})
- The distribution of true effect sizes is addressed by the prediction interval which incorporates true dispersion (T^2) as well as error (V_{M^*})

Prediction Intervals

- Based on sample values

$$LL_{pred} = M^* - t_{df}^{\alpha}\sqrt{T^2 + V_{M^*}}$$

- And

$$UL_{pred} = M^* + t_{df}^{\alpha}\sqrt{T^2 + V_{M^*}}$$

- With

$$df = k - 2$$

$$=\text{TINV}(\alpha, df)$$

Prediction Intervals

- Confidence intervals reflect the accuracy of the mean (in 95% of cases the mean effect size falls in the summary effect)
- The prediction intervals address the dispersion of effects (in 95% of cases the true effect in a new study will fall within the prediction intervals)

Review Questions

1. What information about heterogeneity do prediction intervals provide?
2. How are prediction intervals interpreted?
3. How does this interpretation differ from the interpretation of M^*?

Today’s Second In-Class Activity

- Using “Data Sets for Heterogeneity Analysis XLSX” from the course Website
  - Calculate the prediction intervals (Step 9) of M^* for each data set
  - Interpret the prediction intervals in contrast to M^*