Skin Friction Structures Extracted from Surface Pressure Gradient Field

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Objective

To study the feasibility of extracting skin friction structures from surface pressure gradient field

Mission impossible?
PSP Images on a CRM Wing in Transonic Flow

JAXA Experiments

AoA = 3 deg

AoA = 4.5 deg

AoA = 6 deg
Relation Between Pressure Gradient & Skin Friction

\[ \tau \cdot \nabla p = \mu f_\Omega \quad \text{on } \partial B \]

where

**Boundary Enstrophy Flux (BEF):**

\[
f_\Omega = \mu \partial \Omega / \partial n - 2 \mu \kappa_\omega \Omega
\]

**Boundary Enstrophy:**

\[
\Omega = \left| \omega \right|^2 / 2
\]
Optical Flow Equation in the Image Plane

\[ G + \hat{\tau}_j \frac{\partial p}{\partial x_j} = 0 \]

where \( G = -\mu f \omega \)

Surface pressure gradient and skin friction are intrinsically coupled mainly through a vorticity-dynamic mechanism characterized by a single physical quantity namely the BEF.
Inverse Problem & Variational Solution

The functional with a smoothness constraint:

\[ J(\hat{\tau}) = \int_{\Omega} (G + \hat{\tau} \cdot \nabla p)^2 \, dx_1 \, dx_2 + \alpha \int_{\Omega} \left( |\nabla \hat{\tau}_1|^2 + |\nabla \hat{\tau}_2|^2 \right) \, dx_1 \, dx_2 \]

The Euler-Lagrange equations:

\[ (G + \hat{\tau} \cdot \nabla p) \nabla p - \alpha \nabla^2 \hat{\tau} = 0 \]

where the Neumann condition \( \partial \hat{\tau} / \partial n = 0 \)

If \( p \) \& \( G \) are measured and known,

\[ \hat{\tau} = (\hat{\tau}_1, \hat{\tau}_2) \] can be obtained by solving the E-L equation.
Validation: Falkner-Skan Flow (Wedge Flow)

\[ U(x) = ax^m \]

\[ \pi 2m/(m+1) \]

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**Normalized Pressure**

**Normalized BEF**

**Skin Friction Field**

**Pressure**  **BEF**  **Skin Friction**
Comparison between the Variational Solution & the Falkner-Skan Solution

\[ \tau = \frac{\pi 2m}{m+1} \]
Validation: NACA-0024 Airfoil Flow

Pressure

BEF

Skin Friction

(AoA = 0 deg)
Comparison between the Variational Solution & the Boundary-Layer Solution

![Graphs comparing variational and boundary-layer solutions](image)

- **Variational Solution**
  - $C_f$ vs $x/c$
  - Key features:
    - Zero-pressure gradient
    - Attachment due to transition

- **Boundary-Layer Solution**
  - $C_f$ vs $x/c$
  - Key features:
    - Boundary-layer solution (AoA = 0°)
    - Boundary-layer solution (AoA = 4°)
    - Separation

Note: The graphs illustrate the difference in pressure distribution and flow behavior between the variational and boundary-layer solutions at different angles of attack.
Validation: Junction Flow (Experiments)

Normalized Surface Pressure Field From PSP

Global Luminescent Oil-Film Image
GLOF Measurements in Junction Flow

Skin-Friction Vectors

Skin-Friction Lines

- Separation Line
- Attachment Line
- $S_1$
- $S_2$
- $N_1$
- $N_2$
Validation: Recovered Results from Pressure Gradient Field and BEF Field

The BEF Field

Null

Recovered Skin-Friction Field

Separation Line

Attachment Line

$S_1$

$S_2$

$N_1$

$N_2$
What can we do when the BEF field is not known?

Typical BEF fields
Auxiliary Skin-Friction Field

--- The $\hat{\tau}_p$ -Field

**Definition:**

$$\hat{\tau}_{p,,j} = -\mu^{-1} f_{\Omega}^{-1} \hat{\tau}_i$$

$i, j = 1, 2$

**Equation:**

$$1 + \hat{\tau}_{p,j} \partial p / \partial x_j = 0$$

**Euler-Lagrange Equation:**

$$\left(1 + \hat{\tau}_p \cdot \nabla p \right) \nabla p - \alpha \nabla^2 \hat{\tau}_p = 0$$
The Physical Properties of $\hat{\tau}_p$ -Field

Magnitude: $\|\hat{\tau}_p\| = \|\hat{\tau}\| \mu^{-1} f_\Omega^{-1}$

Direction: $-\nabla p / \|\nabla p\|$

Correspondence

$\|\hat{\tau}_p\| = 0$ Critical Points

Minimum Lines in $\|\hat{\tau}_p\|$ Separation Line or Attachment Line
Junction Flow

The $\| \hat{\tau}_p \|$-Field

The $\hat{\tau}_p$-Lines

Minima

Minima

N_1

S_1

N_2

S_2

N_3
70-deg Delta Wing

PSP Pressure Image

The $\hat{\tau}_p$ -Lines
70-deg Delta Wing

CFD Pressure Image

The $\hat{\tau}_p$ -Lines

BEF Field
70-deg Delta Wing

CFD Pressure Gradient Magnitude

The $\hat{\tau}_p$ -Vectors

The $\|\hat{\tau}_p\|$ -Field
70-deg Delta Wing

CFD Near-Wall Streamlines

The $\tau_p$ -Lines
70-deg Delta Wing

The $\hat{\tau}_p$ -Lines

$\nabla p \perp$ Separation Line
Summary

- Given the $\nabla p$ field & BEF field,
  - The $\hat{\tau}$ field
    - Complete skin-friction topology

- Given the $\nabla p$ field alone,
  - The $\hat{\tau}_p$ field
    - Possible critical points & separation/attachment lines
Ad-hoc, Approximate Approach for Flow over a Relatively Flat Surface: 
Skin-Friction Decomposition

\[ \tau = \tau_0 + \tau_p \]

\( \tau_p \): ‘Skin friction from surface pressure gradient’
a generic skin friction structure associated with \( \nabla p \)

Examples: pipe flows & Couette flow

\( \tau_0 \): ‘Complementary part’
associated with the non-\( \nabla p \)-driven flow field

Examples: flat-plate boundary layer
70-deg-Delta Wing
(M = 0.55, Po = 100 kPa, AoA = 20 deg)

\( \tau_p - field \)

\( \tau_p - lines \)
70-deg-Delta Wing
(M = 0.55, Po = 100 kPa, AoA = 20 deg)

\[ \tau_0 + \tau_p - \text{field} \]

\[ \tau_0 + \tau_p - \text{lines} \]
Transonic Flow over a CRM Wing

$\tau_p - \text{vector field}$

AoA = 6 deg
Transonic Flow over a CRM Wing

\[ \tau_0 + \tau_p - \text{vector field} \]

AoA = 6 deg