Computer Vision and Measurements in Aerospace Applications

Tianshu Liu

Department of Mechanical and Aeronautical Engineering
Western Michigan University
Kalamazoo, MI 49008
Objective

To build a unified theoretical framework for quantitative image-based measurements of morphology and motion fields of deformable bodies like fluids

• Geometric Structures:
  Points, Curves, Surfaces

• Motion Fields:
  Points, Curves, Surfaces (Geometric Flow)
  Complex Continuous Patterns
Background

- **Photogrammetry**
- **Computer Vision**

**Human Vision** ↔ **Computer Vision**

Analogy
Optical Info Processing

Category-Based and Higher-Level Processing
Recovery of the functional properties of objects from their geometric, radiometric, physical, and dynamical properties. Typical techniques include neural networks, fuzzy logic, genetic algorithms, machine reasoning, knowledge-based expert systems, statistical modeling, etc.

Three-Dimensional Data Processing
- **Geometry:**
  - surface-based (2.5D) and volumetric (3D) representations and constructions
- **Radiometry:**
  - mapping radiometric and extracted physical properties onto 2.5D and 3D objects
- **Motion:**
  - motion fields of 2.5D and 3D objects

Geometry- and Physics-Based Image Processing
- **Geometry:**
  - edges, lines, texture, shapes, positions, areas, orientations, scales, patterns, fractals, etc.
- **Radiometry:**
  - radiation intensity, color, spectroscopic imaging, fluorescence imaging, radar imaging, acoustic imaging, physical properties extracted from radiometric images such as temperature, pressure, species, etc.
- **Motion:**
  - rigid body motion, motion of deformable bodies (such as fluids, and elastic and viscoelastic matters), transport phenomena, pattern formation, fractal dynamics, etc.

Initial Image Processing
Various transforms and filterings

Images
Geometric and Physical-Based Processing

- Emission Radiance in Images
- Geometric Features In Images
- Perspective Transformation
  - Mapping of Geometry and Motion
  - Mapping of Physical Quantities
- Geometrical, Kinematical and Physical Representations in 3D object Space
- Principles of Physics
- Mathematical Models

Principles of Physics
Mathematical Models
Geometric and Physical-Based Processing
Geometric Features In Images
Perspective Transformation

Emission Radiance in Images
Geometric Features In Images
Important Quantities in Aerodynamics

(1) Pressure  (2) Temperature  (3) Skin Friction  
(4) Velocity Field  (5) Attitude and Kinematics  
(6) Shape

Molecular Sensors!
Combination of Approaches

• Perspective Geometry
• Differential Geometry
• Continuum Kinematics & Dynamics
• Radiometry

Needs for New Methods

Quantitative, Dynamic, Universal, Applicable to Complex Patterns & Motions of Deformable Bodies
Relevant Topics

(1) Perspective Projection Transformation
(2) Projective Developable Conical Surface
(3) Perspective Projection under Surface Constraint
(4) Perspective Projection of Motion Field Constrained on Surface
(5) The Correspondence Problem
(6) Composite Image Space and Object Space
(7) Perspective Invariants of 3D Curve
(8) Reflection and Shape Recovery
(9) Motion Equations of Image Intensity
Perspective Projection Transformation

Camera parameters

$$\Pi = (\omega, \phi, \kappa, X_c^1, X_c^2, X_c^3, x_p^1, x_p^2, K_1, K_2, P_1, P_2)$$
Formulations of Perspective Transformation

(1) Collinearity Equations

\[ x^1 - x_p^1 + \delta x^1 = -c \frac{m_1^T (X - X_c)}{m_3^T (X - X_c)} \]

\[ x^2 - x_p^2 + \delta x^2 = -c \frac{m_2^T (X - X_c)}{m_3^T (X - X_c)} \]

(2) Homogenous Coordinate Form

\[ x_h = \lambda P_h X_h \]

(3) W-Vector Form

\[ W_1^T (X - X_c) = 0 \]

\[ W_2^T (X - X_c) = 0 \]
Geometric Image Measurements

Projection

3D → 2D

Recover the Lost Dimension
Perspective Developable Conical Surface

Plane parallel to image

3D curve

Ray Equation

\[ X_{c_p} - X_c = \lambda^{-1} ( P x_{h0} + \int_{0}^{s} P_{32} t \, ds ) \]

Conical Surface Equations

\[ ( X - X_c ) \cdot N_D(s) = 0 \]

\[ ( X - X_c ) \cdot dN_D(s) / ds = 0 \]
Reconstruction of 3D Curves and Surfaces Using Projective Conical Surfaces

3D Space Curve

Surface
Reconstruction of 3D Displacement Vectors

Providing a rational and general method for Stereoscopic Particle Image Velocimetry (SPIV) and Scalar Image Velocimetry (SIV)
Motion Field of 3D Space Curve

Variational Problem:

\[ \| \Delta_{s\epsilon}X - U(X)\Delta t \| \rightarrow \min \]

Physical and Geometric Constraints:

\[ G_i[U(X)] = 0 \quad i = 1, 2, \ldots \]
Perspective Projection under Surface Constraint

\[ X^3 = F(X^1, X^2) \]

One-to-One Differential Relation

\[
\begin{pmatrix}
\frac{dX^1}{dX^2}
\end{pmatrix} = m^T_3 (X_c - f_s)Q^{-1}
\begin{pmatrix}
\frac{dx^1}{dx^2}
\end{pmatrix}
\]

Geometric Structures

\[ dS^2 = |dX| = g_{\alpha\beta} dx^\alpha dx^\beta \]

**Quantities**: tangents, normal, length, angle, area, topology
Surface-Constrained Motion Field

\[ u = \frac{d}{dt} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \frac{Q}{m_3^T (X_c - f_S)} \begin{pmatrix} U_1[ f_S(x) ] \\ U_2[ f_S(x) ] \end{pmatrix} \]

Optical Flow

Motion Field
The Point Correspondence Problem

Generalized Longuet-Higgins Relation:

\[
(x^\alpha_{h(2)} + \delta x^\alpha_{h(2)}) Q_{\alpha\beta} (x^\beta_{h(1)} + \delta x^\beta_{h(1)}) = 0
\]
Determining Point Correspondence

Four Images or Cameras

Six Longuet-Higgins Equations for Six Unknowns
Composite Image Space and Object Space

One-to-One Relation

Reconstruct 3D Displacement Vectors from Composite Image Coordinates
Perspective Invariants of 3D Curve

Torsion

\[ \frac{\tau_{im,1} d_{12}^2}{\tau_{im,2} d_{21}^2} = \frac{\tau_{obj,1} D_{12}^2}{\tau_{obj,2} D_{21}^2} \]

Curvatures

\[ \frac{\kappa_{im,2} d_{12} i^2(1,1',2,3)}{\kappa_{im,1} d_{21} i^2(1,2,2',3)} = \frac{\kappa_{obj,2} D_{12} I^2(1,1',2,3)}{\kappa_{obj,1} D_{21} I^2(1,2,2',3)} \]

Distances

\[ \frac{d_{21} d_{43}}{d_{41} d_{23}} = \frac{D_{21} D_{43}}{D_{41} D_{23}} \] (Brill, et al. 1992)

Geometrical Flow Problem

Rectifying plane

Osculating plane
\[ I( x ) = c_{sys} \rho_a E_a \]
\[ + c_{sys} E_{ls} \left[ \rho_d N \cdot L_s + \rho_s p( a_N N \cdot V + a_L L_s \cdot V ) \right] \]
Motion Equations of Image Intensity

Image Intensity and Radiance

\[ I( x, t ) = c \, L( X, a, g, t ) \]

Physical parameters: \( p = ( p_1, p_2, \ldots, p_N )^T \)

Geometric parameters: \( q = ( q_1, q_2, \ldots, q_M )^T \)

Radiance: \( L( X, p, q ) \)
Generic Motion Equations of Image Intensity

\[
\frac{\partial I}{\partial t} + \mathbf{u} \cdot \nabla_x I = c_{sys} \left( \frac{\partial L}{\partial t} + \mathbf{U} \cdot \nabla_x L + \frac{dp}{dt} \cdot \nabla_p L + \frac{dq}{dt} \cdot \nabla_q L \right)
\]

Optical Flow \quad \rightarrow \quad Velocity Field

Physical parameters: \quad \mathbf{p} = (p_1, p_2, \cdots, p_N)^T

Geometric parameters: \quad \mathbf{q} = (q_1, q_2, \cdots, q_M)^T

Radiance: \quad L(X, p, q)
Emitting Passive Scalar Transport

**Governing Equation**

\[
\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + U \cdot \nabla \psi = D_{\psi} \nabla^2_x \psi
\]

**Radiance**

\[
L(X,t) = c_{\psi} \psi(X,t)
\]

Perspective Projection onto Image Plane
Motion Equation of Image Intensity for Emitting Passive Scalar Transport

\[ \frac{\partial I}{\partial t} + u_\alpha \frac{\partial I}{\partial x^\alpha} = D_\psi \left( h_\lambda \frac{\partial I}{\partial x^\lambda} + h_{\lambda\alpha} \frac{\partial^2 I}{\partial x^\alpha \partial x^\lambda} \right) \]

Optical Flow and Velocity Field on Surface

\[ u = \frac{d}{dt} \begin{pmatrix} x^1 \\ x^2 \end{pmatrix} = \frac{G}{m_3 \cdot (X_c - f_S)} \begin{pmatrix} U_1[f_S(x)] \\ U_2[f_S(x)] \end{pmatrix} \]
Governing Equation

\[
\frac{d\psi}{dt} = \frac{\partial \psi}{\partial t} + U \cdot \nabla \psi = D_\psi \nabla^2_x \psi
\]

\[
L = L_0 \exp\left( - \int_0^s \beta_{\text{ext}} \, ds \right)
\]

Perspective Projection onto Image Plane
Motion Equation of Image Intensity for Light Transmitting Scalar Transport

\[ \frac{\partial I}{\partial t} + u_\beta \frac{\partial I}{\partial x^\beta} = D_\psi \lambda^2 \left( \frac{\partial^2 I}{\partial x^\beta \partial x^\beta} - I^{-1} \frac{\partial I}{\partial x^\beta} \frac{\partial I}{\partial x^\beta} \right) \]

Optical Flow and Path-Averaged Velocity

\[ u_\alpha \frac{\partial I}{\partial x^\alpha} = <U_{12}>_\psi \cdot \nabla_{12} I \]

where \[ <U_{12}>_\psi = \frac{\int_{\Gamma_1}^{\Gamma_2} \psi U_{12} \, d\bar{X}^3}{\int_{\Gamma_1}^{\Gamma_2} \psi \, d\bar{X}^3} \]
Schlieren Image of Density-Varying Flows

**Image Intensity and Density Gradient**

\[
\frac{I - I_K}{I_K} = C_{schl} \int_{\Gamma_1}^{\Gamma_2} \frac{\partial \rho}{\partial X^2} \, dX^3
\]

**Motion Equations of Image Intensity**

\[
\frac{\partial}{\partial t} \int_{X_0^2}^{X^2} (I - I_K) \, dX^2 + \nabla_{12} \cdot [\langle U_{12} \rangle \rho \int_{X_0^2}^{X^2} (I - I_K) \, dX^2 ] = 0
\]

**where**

\[
\langle U_{12} \rangle \rho = \frac{\int_{\Gamma_1}^{\Gamma_2} \rho U_{12} \, d\bar{X}^3}{\int_{\Gamma_1}^{\Gamma_2} \rho \, d\bar{X}^3}
\]

\[
\nabla_{12} = \left( \frac{\partial}{\partial X^1}, \frac{\partial}{\partial X^2} \right)^T
\]
Shadowgraph Image of Density-Varying Flows

Image Intensity and Second-Order Density Derivative

\[
\frac{I - I_T}{I_T} = C_{shad} \int_{\Gamma_1}^{\Gamma_2} \nabla_{12}^2 \rho \, dX^3
\]

Motion Equations of Image Intensity

\[
\frac{\partial}{\partial t} \nabla_{12}^{-2} (I - I_T) + \nabla_{12} \cdot [<U_{12}> \rho \, \nabla_{12}^{-2} (I - I_T)] = 0
\]

where \( \nabla_{12}^2 \phi = I - I_T \) solution \( \nabla_{12}^{-2} (I - I_T) \)
Transmittance Image of Density-Varying Flows

Image Intensity and Density

\[
\frac{I - I_T}{I_T} = C_{\text{trans}} \int_{\Gamma_1}^{\Gamma_2} \rho \, dX^3
\]

Motion Equations of Image Intensity

\[
\frac{\partial}{\partial t} (I - I_T) + \nabla_{\mathbf{I}_2} \cdot [\langle \mathbf{U}_{\mathbf{I}_2} \rangle \rho (I - I_T)] = 0
\]
Typical Applications

• Aerodynamic measurements: pressure and temperature sensitive paints, videogrammetric attitude measurement, stereoscopic PIV, laser-tagging technique, schlieren, shadow and transmittance imaging, oil-film/liquid-crystal skin friction measurements.

• Metrology and kinematics of large inflatable space structure.
Unification of Measurement Systems

Conventional Techniques

- Force Balances
- Accelerometers
- Hot-Wires & Films
- Pressure Taps & Probes
- Temperature & Heat Transfer Gauges

Image-Based Techniques

- PSP & TSP
- Videogrammetry
- Global Velocimetries (PIV, DGV)
- Schlieren/Shadowgraph
- Oil & LC - Film Skin Friction Meters
Data Fusion and Understanding

**Measurements**
- Integrated data & data at discrete locations
- Distributions on surfaces
- Fields in 3D space

**Data Base & Models**
- Aerodynamics data base
- Theoretical models
- CFD

“SuperAerodynamicist”: Intelligent Expert System!?
Conclusions

We will see unified image-based instrumentation providing non-contact, global measurements of important physical, geometric and dynamical quantities in wind tunnel testing.

**Ideal Aerodynamics Lab**

- Unified Measurement Techniques
- Wind Tunnel Testing
- CFD Theories
- Aerospace System Design