Global Skin Friction Diagnostics Based on Surface Mass-Transfer Visualizations

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Objective

To explore the feasibility of global skin friction diagnostic based on surface mass-transfer visualizations

Current State

(1) Global luminescent oil-film skin friction meter

(2) Global skin friction diagnostics based on surface heat-transfer/temperature visualizations
The Specific Question Came from an Image

Pyrene PSP image on a 75-deg delta wing in a large wing tunnel at ONERA (Bouvier, Le Sant & Merienne 2001)

The pattern results from sublimation of Pyrene PSP

**Question:**
How to extract a skin friction field from this image?
The Issues in Global Skin Friction Diagnostics Based on Mass-Transfer Visualizations

(1) *There is no explicit analytical solution for the relation between surface species density, mass flux and skin friction vector.*

(2) *It was not recognized that it should be solved globally as an inverse problem.*
Asymptotic Form of Binary Mass Diffusion Equation at Wall

\[ F + \tau_i \frac{\partial \omega_{1w}}{\partial X_i} = 0 \]

where

\[ F = \frac{\mu}{\rho_w D_{12}} \left( -\frac{\partial \dot{m}_{1w}}{\partial t} + D_{12} \frac{\partial^2 \dot{m}_{1w}}{\partial X_i \partial X_i} \right) \]

**Relevant Quantities:**

- \( \tau_i \) **Skin friction vector (to be determined)**
- \( \omega_1 = \frac{\rho_1}{\rho} \) **Surface density of species 1 (measurable)**
- \( \dot{m}_{1w} = -D_{12} \rho_w (\frac{\partial \omega_1}{\partial X_3})_w \) **Surface mass flux (measurable)**
Snapshot Solution in a Short Interval or Window

\[ \langle F \rangle_t + \tau_i \frac{\partial \omega_{1w,s}}{\partial X_i} = 0 \]

where

\[ \langle F \rangle_t = \frac{\mu}{\rho_w D_{12} \Delta T} \left( -\dot{m}_{1w,s} + D_{12} \Delta T \frac{\partial^2 \dot{m}_{1w,s}}{\partial X_i \partial X_i} \right) \]

**Projection onto Image Plane**

**Projected skin friction vector**

\[ \hat{\tau}_j \propto h_{ji} \tau_i \]

where

\[ h_{ji} = \frac{\partial F_j}{\partial X_i} \]

define the directions of the coordinate curves on a surface
Generic Optical Flow Equation for Skin Friction in Mass-Transfer Visualizations

\[ G + \hat{\tau}_j \frac{\partial g}{\partial x_j} = 0 \]

**PSP Visualization**

\[ g = \frac{I_{\text{ref}}}{I} \quad G = g - \frac{I_{\text{ref}}}{I_{00}} + \varepsilon_0 \]

**Sublimation Visualization with Changing Density**

\[ g = \frac{I}{I_{\text{ref}}} \quad G = g - 1 + \varepsilon_1 + \varepsilon_2 \]

**Sublimation Visualization with Changing Thickness**

\[ g = \frac{I}{I_{\text{ref}}} \quad G = -g + 1 + \varepsilon_3 \]
Variational Formulation

The functional with a smoothness constraint:

\[ J(\hat{\tau}) = \int_{\Omega} \left( G + \hat{\tau} \cdot \nabla g \right)^2 \, dx_1 \, dx_2 + \alpha \int_{\Omega} \left( |\nabla \hat{\tau}_1|^2 + |\nabla \hat{\tau}_2|^2 \right) \, dx_1 \, dx_2 \]

The Euler-Lagrange equations:

\[
\left[ G + \hat{\tau} \cdot \nabla g \right] \nabla g - \alpha \nabla^2 \hat{\tau} = 0
\]

where the Neumann condition \( \partial \hat{\tau} / \partial n = 0 \)

If \( g \) and \( G \) are measured and known,

\( \hat{\tau} = (\hat{\tau}_1, \hat{\tau}_2) \) can be obtained by solving the E-L equation.
Uncertainty Analysis and Intrinsic Limitation

The relative error in skin friction calculation:

\[
\left( \frac{\delta \hat{\tau}}{\| \hat{\tau}_0 \|} \right)_N = - \frac{\delta G}{\| \nabla g_0 \| \| \hat{\tau}_0 \|} - \left( \frac{\hat{\tau}_0}{\| \hat{\tau}_0 \|} \right) \cdot \partial \mathbf{N}_T + \frac{\alpha}{\| \nabla g_0 \|^2} \nabla^2 \left[ \left( \frac{\delta \hat{\tau}}{\| \hat{\tau}_0 \|} \right)_N \right]
\]

Consequences:

1. When \( \| \nabla g_0 \| \to 0 \) the error blows up \( (\delta \hat{\tau})_N / \| \hat{\tau}_0 \| \to \infty \)

which imposes an intrinsic limitation on this method

2. The Lagrange multiplier \( \alpha \) must be as small as possible.
Unsteady Measurements with Fast PSP Visualization

Superposition

\[ \hat{\tau}_j = \hat{\tau}_{j,s} + \hat{\tau}'_j \]

**Quasi-steady solution:**

\[ G_1 + \hat{\tau}_{j,s} \frac{\partial g_s}{\partial x_j} = 0 \]

where \( G_1 = g_s - g_{00} + \langle \epsilon \rangle_t \)

**Variation for unsteady effect:**

\[ G_2 + \hat{\tau}'_j \frac{\partial g_s}{\partial x_j} = 0 \]

where \( G_2 = \Delta g_{k+1} - \Delta g_k + \hat{\tau}_{j,s} \frac{\partial (\Delta g_{k+1} / 2)}{\partial x_j} \)
Circular Impinging Nitrogen Jet Visualized with PSP

- PSP: Ru(phen) in GE RTV 118
- UV for illumination
- CCD camera with long-pass filter
- 75 microns Mylar

\[
D = 2 \text{ mm} \\
U_o = 50 \text{ m/s} \\
Re_D = U_o D/\nu = 6.4 \times 10^3 \\
H/D = 6
\]
Normal Impinging Nitrogen Jet

Normalized PSP Intensity Distribution

Normalized skin friction field

Intensity Ratio $I/I_{\text{ref}}$

Skin Friction Vectors and Normalized Magnitude
Transverse Distributions of Skin Friction Magnitude for Normal Impinging Jet

![Graph showing normalized skin friction magnitude vs. x/H for different methods and tests, including Mass-Transfer Method and Hot Film tests with theoretical solutions.]
75-deg Oblique Impinging Jet

Normalized PSP Intensity Distribution

Normalized skin friction field
Transverse Distributions of Skin Friction Magnitude for 75-dge Oblique Impinging Jet

![Graph showing normalized skin friction magnitude vs. x/H for a 75-degree oblique impinging jet. The graph includes data points and a trend line representing mass transfer method and hot film measurements.](image-url)
Dual Colliding Impinging Nitrogen Jets Visualized with PSP

- **PSP:** Ru(phen) in GE RTV 118
- UV for illumination
- CCD camera with long-pass filter
- 75 microns Mylar

\[ D = 5.18 \text{ mm} \]
\[ \phi = 30^\circ \]
\[ H = 29.5 \text{ mm} \]
\[ U = 4.14 \text{ m/s} \]
PSP Intensity Images at Different Offsets
Skin Friction Vector and Magnitude Fields
Skin Friction Lines

Two sets of diagrams illustrating skin friction lines in a flow field. The axes are labeled as $y/H$ and $x/H$, with ranges from -0.4 to 0.4. The diagrams depict the distribution of skin friction along the boundary of a flow, showing how the friction force varies across different sections of the boundary.
Oscillating Impinging Nitrogen Jets at 9.4 kHz Visualized with Fast PSP (Gregory et al. 2007)
Quasi-Steady and Variation Fields at 0 μs

Quasi-steady field

Variation field associated with unsteady effect (~ 10%)
Unsteady Skin Friction Fields Reconstructed by Superposition of Quasi-Steady and Variation Fields
Sublimation Visualization with Pyrene PSP on a 75-deg Delta Wing (ONERA)

**Sublimation image**

Intensity Ratio $I/I_{\text{ref}}$

**Extracted skin friction field**

Skin Friction Lines

- Primary attachment line
- Secondary separation line
- Secondary attachment line
Sublimation Visualization with Acenaphthene in Shock-BL Interaction at Mach 6 (VKI)

Sublimation image

Extracted skin friction field
Conclusions

• **Global skin friction diagnostics is feasible based on surface mass-transfer visualizations.** In particular, it is possible for unsteady skin friction diagnostics with fast PSP.

• **The intrinsic limitation is that this method has large error when the intensity gradient is zero.**

• **It is incorporated into the unified framework of physics-based optical flow method for global velocity and skin friction diagnostics.**